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DESIGN AND OPTIMISATION OF
AN ULTRASONIC DIE SYSTEM
FOR FORMING METAL CANS.

by
Christopher Francis Cheers M.A.

A doctoral thesis submitted in partial
fulfilment of the requirements for the
award of Doctor of Philosophy of the
Loughborough University of Technology

June 1995

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"It is an important and popular fact that things are not always as they seem. For instance, on the planet Earth, man had always assumed he was more intelligent than dolphins because he had achieved so much - the wheel, New York, wars and so on - whilst all the dolphins had ever done was muck about in the water having a good time. But conversely, the dolphins had always believed they were far more intelligent than man - for precisely the same reasons."

Douglas Adams

The Hitch Hiker's Guide to the Galaxy

1979
ABSTRACT

A new manufacturing process has been developed for reducing the diameter of one end of a tinplate can by over 30%. Conventional processes are limited to a maximum of 10% reduction and typically operate at less than 5%. The improvement was achieved by using special tooling and ultrasonic excitation of the die to reduce the forming force.

Ultrasonics have been used in this way before but without a full understanding of the numerous modes of vibration of the die, and how they interact, the efficiency of earlier systems was low. Finite element analysis has been used to characterize the natural modes and frequencies of radial-mode ultrasonic dies and this has led to the development of highly efficient systems. In special cases a non-round die has been required to overcome undesirable modal characteristics; optimum shapes have been developed. A completely new method of mounting the ultrasonic dies was designed and its geometry optimized (again using finite element analysis) to further improve the efficiency of the system.

The new system operates at an amplitude under load approximately three times greater than the earlier equipment. The reduction in forming force (between 30 and 60%) makes the difference between success and failure for the manufacturing process.
ACKNOWLEDGEMENTS

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Principles and results of ESPI analysis described in section 6.3, including figures 6.08 to 6.16 are reproduced courtesy of Dr John Tyrer and Dr Mike Shellabear.
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This project concerns the development of a new technique for forming metal aerosol cans using ultrasonic vibrations to assist the forming process. The work described forms part of a collaborative research project between CarnaudMetalbox and Loughborough University sponsored by the Science and Engineering Research Council under the specially promoted programme for the development of high speed machinery. Research at Loughborough included the development of special optical and modal analysis techniques for vibration measurement and design optimisation, while at Wantage finite element analysis was used to develop the design of new ultrasonic tools. For a general overview of project aims and achievements see Chapman and Tyrer [1].

The project involves two main areas:

1) Aerosol can manufacturing methods

2) High power ultrasonics, particularly where applied to metal forming processes.

In this chapter the reasons for the use of ultrasonics are explained and possible reasons for their beneficial effects discussed. Safety considerations when working with high power ultrasonics are also included.

A short summary of the thesis follows to indicate the content of the remaining chapters.
1.1 BACKGROUND

The background to the project includes the manufacture of aerosol cans (which was the first application of the technique), and the use of high power ultrasonics.

1.1.1 Background - Aerosol Cans

Most aerosol cans available at present are manufactured from either steel or aluminium. The production method depends on the material. Aluminium cans are produced by extrusion of a cylinder with a closed end, followed by multiple die forming operations to reduce the diameter of the open end and create the rolled end on which the valve assembly is fitted. Figure 1.01 shows diagrammatically the whole process while figure 1.02 shows a series of cans at each stage of the neck reduction.
Steel (tinplate) cans are produced by a different process shown in figure 1.03. A rectangular shape is cut from sheet, rolled into a cylinder and welded along the seam. The round end pieces (pressed from another sheet of steel) are then fitted by a clinching process known as double seaming.

Both types of can are supplied to the filler (cosmetics / pharmaceutical companies) with the top open. Valve assemblies are supplied separately. The filler puts his product plus propellant into the can and then fits a valve assembly which is fixed by clinching (figure 1.04). To ensure that the filling operation goes smoothly and to avoid leakage after filling both the can and the valve assembly must be made to tightly controlled dimensions - typical tolerances are ± 0.1 mm.
1. Introduction

**Figure 1.03** - Manufacturing Process for Tinplate Aerosol Cans

A rectangular is rolled into the ends are blank of tinplate... while end components are formed from another sheet of tinplate...

**Figure 1.04** - Fitting of Aerosol Valve Assembly

Valve assembly comprising:
- Nozzle
- Valve cup
- Valve (clinched in valve cup)
- Tube

Fitted to can sealing compound or gasket prevents pressure leakage.

Valve cup is swaged out underneath curl to fix it in place.
1. Introduction

Each can material has advantages and disadvantages. The advantages of aluminium cans are their smooth, attractive appearance and the reduced risk of corrosion (many aerosol products are corrosive). The main advantage of steel is its lower cost - the raw material cost of aluminium is relatively high and aluminium can walls must be thicker to withstand the same internal pressure (the yield strength of the aluminium used is lower than that of steel). Also steel aerosol cans may be decorated with a higher quality design because printing is done on the flat sheet - this is easier than printing onto a cylindrical can surface. To counteract the risk of corrosion of steel cans various systems are used to apply a protective coating to the inside and outside surfaces.

Most aerosol cans on sale in the UK are made from steel, primarily because of their lower cost. Aluminium cans are used mainly in small sizes for cosmetic products where the appearance advantages outweigh the extra cost.

CarnaudMetalbox Aerosols UK manufactures only tinplate aerosol cans. To gain a competitive advantage, therefore, it was desirable to improve the design of the tinplate aerosol to make it more competitive with the aluminium can. This was the basis for the development of the "coneless" tinplate aerosol can, so called because the top component (the cone) has been eliminated. The rolled form on which the valve fits must be formed from the cylindrical body as in the aluminium can. This has the advantage of an attractive appearance similar to the aluminium can while maintaining the lower cost of the tinplate can (in fact the cost of a coneless tinplate can may even be less than that of the conventional can because the cost of producing the cone is eliminated). Figure 1.05 shows the manufacturing process for coneless tinplate aerosols. Figure 1.06 shows for comparison a conventional tinplate aerosol, the new coneless aerosol with and without a shrink-sleeve decoration (to disguise the unprinted weld area) and an aluminium aerosol.
1. Introduction

A rectangular "BLANK" of tinplate... is rolled into a cylinder and welded along the overlap... the top end is necked and curled to the cone shape... the base is spin-necked and flanged and the base component is formed from another sheet of tinplate...

**FIGURE 1.05 - MANUFACTURING PROCESS FOR "CONELESS" TINPLATE AEROSOLS**

**Figure 1.06** Comparison of different types of aerosol can
The process of forming the end shape from the cylinder is much more difficult in steel than in aluminium, because the steel is harder, thinner and more liable to work hardening than the aluminium material. This means that even more forming operations are required to achieve any given reduction in diameter. A similar process has been used on steel cans by Stoffel [2] but in this case the rolled end on which the valve fits was formed by rolling the material inwards. This makes the necking operation much easier and reduces work hardening of the can material but leaves the raw edge of the material inside the can. This part of the can is particularly prone to corrosion because it lacks the protection of the layers of tin and lacquer that cover the surfaces of the can walls. This means that a steel aerosol manufactured in this way would be limited to non-corrosive products, such as solvent-based hair sprays. Water-based products such as mousses and shaving foam (for which there is a large market) could not use such a container and it is probably for this reason that commercial exploitation has not followed.

The problem is essentially that the conventional die necking process is incapable of reducing the can diameter by more than a few millimetres in each operation. If a greater reduction is attempted then the material undergoes a hoop buckling failure known as "pleating" (figure 1.07). The basis of the new necking operation is to prevent this by using a shaped plug inside the die (figure 1.08). The profile of the die and plug match, so that the gap between them is about 1.5 times the material thickness. This is sufficient to permit the material to pass through, even if it thickens slightly, but not to permit the material to pleat.
1. Introduction

FIGURE 1.07 PLEATING (HOOP BUCKLING) DURING NECKING

FIGURE 1.08 - SEQUENCE OF EVENTS - DIE NECKING WITH INTERNAL PLUG
Using this type of tooling the problem of pleating is eliminated, so far greater reductions are made possible (More than 30% reduction of diameter has been achieved in a single operation, compared to about 5% maximum in conventional necking processes). The achievable reduction is still limited, however, by the force that can be applied. The can must be pushed into the die and the force applied to the base must be transmitted through the body of the can to the neck area (see figure 1.08). Thus the maximum force that can be applied is limited by the strength of the can body. If the necking force exceeds the strength of the can body then the necking will stop and the can will be crushed. Figure 1.09 shows a can crushed by excessive forming force.
1.1.2 Analysis of can body strength

The strength of the can body depends on a number of factors including the Young's modulus and yield stress of the material, the plate thickness and the can diameter. The mode of failure may be buckling or yield, whichever occurs at the lower level of applied force. The yield strength of the can is easily calculated:

\[ F_y = 2\pi rt\sigma_y \]

where \( F_y \) = yield force
r = can radius 
\( t = \text{can thickness} \)

i.e. Yield force is yield stress times area. 
\( \sigma_y = \text{yield stress} \)

The formula for buckling collapse is more complicated. Roark [3] page 689 gives the following for critical stress in a thin walled cylinder:

\[
\sigma' = \frac{1}{\sqrt{3}} \cdot \frac{E}{\sqrt{1-\nu^2}} \cdot \frac{t}{r}
\]

where \( \sigma' = \text{critical stress} \)
\( E = \text{Young’s Modulus} \)
\( \nu = \text{Poisson’s ratio} \)

but suggests that measured values are typically between 40 and 60% of this theoretical value. Taking 40% and multiplying by the cross sectional area gives the predicted collapse load in buckling:

\[
F' = 0.4619 \frac{\pi t^2}{\sqrt{(1-\nu^2)}} \cdot E
\]

where \( F' = \text{buckling force} \)

Into these formulae we can put the known values for the can geometry and the material properties of the steel as follows:

\( E = 210 \text{ GPa}, \sigma_y = 240 \text{ MPa}, r = 22.5 \text{ mm}, t = 0.21 \text{ mm}, \nu = 0.3 \)

The result is a calculated yield load of 7.1 kN, and a buckling load of 14.1 kN. This suggests that the mode of failure for this type of can will be yield. The measurements described in section 6.5 confirm this, and the measured collapse force matches this calculated value well.

The mode of failure and the formula describing the collapse load are important to determine what could be done to improve the process. Theoretical analysis of the neck forming (appendix 1) indicates that the forming force is proportional to yield stress, diameter and material thickness (and is also dependant on the amount of reduction, the work hardening characteristics of the material and friction). Since both the forming force and the strength of the can are proportional to the same three parameters, varying these parameters will not improve the forming process or permit a greater reduction. This can be achieved only by reducing friction or by reducing work hardening of the material.
In fact this is oversimplifying the situation slightly. The force required to form the neck depends on the yield stress in the hoop direction, while the can strength depends on the yield stress in the axial direction. The tinplate sheet is not isotropic - the rolling process imposes a "grain direction" that is visible to the naked eye. The nature of the anisotropy varies from one material to another but for the type of tinplate used for this product ("temper 2") the yield stress is about 5% less along the grain than across it. The 45 mm diameter aerosol cans are usually made with the grain direction along the can (called "H-grain") in order to maximise the efficiency of material usage within the sheet sizes available from tinplate suppliers. This is not ideal because the higher yield stress in the hoop direction increases the forming force while the lower axial yield stress reduces the strength of the can body. Making the cans with the grain direction around the can ("C-grain") improves the process. To manufacture 45 mm "coneless" aerosols C-grain material was specified. The small cost penalty associated with this is far outweighed by the improved neck forming.

Apart from the (marginal) issue of grain direction, there are three methods of maximising the achievable necking reduction:

1) Minimise work hardening
2) Minimise friction
3) Radically change the nature of the forming process.

It was thought that the use of ultrasonics might help to achieve some or all of these aims.

1.1.3 Background - Ultrasonics

Ultrasonics can be defined as mechanical vibrations in a solid or fluid at a frequency higher than the range audible to humans. The lowest ultrasonic frequency is normally taken as 20 kHz. The top end of the frequency range is
limited only by the ability to generate the signals - frequencies in the gigahertz range have been used.

Historically ultrasonic vibrations have been used for a huge variety of applications. These can mostly be divided into two broad categories: low power ultrasound (up to about ten watts) and high power ultrasound (ranging from hundreds of watts to tens of kilowatts). Low power applications include non-destructive material testing (particularly for welds), fluid level measurement, thickness measurement and medical imaging. These applications are not highly relevant to the subject of this research and so will not be discussed in detail. High power industrial applications of ultrasound include welding of metals and plastics, ultrasonic cleaning and sonochemistry (altering rates and/or products of chemical reactions). Some of these are relevant to this research because there are common problems in generating, transmitting and controlling the ultrasonic vibrations. For a general review of these and many other applications of high power ultrasonics the reader is referred to a very comprehensive review by Perkins [4]. For more detail Vigoureux [5] gives a detailed account of the physics of high-power ultrasound while Puskar [6] gives a comprehensive description of experimental work in this field, particularly in Eastern Europe where much of the research has been conducted.

A common feature of all high power applications is the use of frequencies at the lower end of the scale (i.e. in the range 20 to 60 kHz). This is because the power available is limited by mechanical stress in the vibrating parts (as described further in section 3.8). Conversely higher frequencies (and square waves or step functions that include high frequency harmonics) tend to be used in measuring applications because the shorter wavelength offers greater accuracy, and at low power mechanical stress is not a problem.

In the following sections the application of high power ultrasound to metal forming is discussed, along with methods of generating the ultrasonic vibrations and the benefits obtainable.
1.1.4 Application of high power ultrasound to metal forming.

The history of research into the use of ultrasound in metal forming is discussed in section 1.3.1. The technique has been assessed in diverse operations including wire drawing, tube drawing, deep drawing, wall ironing and necking, also called nosing or upsetting (i.e. reducing the diameter of one end of a cylinder). In general the use of ultrasound has led to some reduction in the forming force and/or increase in the maximum strain achievable. Note that a reduction in forming force is not, in itself, particularly useful, but an increase in maximum strain can have a dramatic effect because this may permit the use of fewer operations to manufacture a product or even make possible a product that otherwise could not be formed.

The application most similar to the work described in this thesis was reported by Skachko, Pashchenko et al [7], [8]. This group applied high-power ultrasonics to the forming of an aerosol neck, and found that it was possible to reduce the number of operations required from 9 to 3 using ultrasonic vibrations in the axial and/or the radial directions. It is likely that the number of operations could not be further reduced because of hoop buckling (pleating) as described in section 1.1.

This problem would have been much more severe had the can material been thinner and harder as a steel can is. In this case the use of a shaped plug is essential to prevent pleating. Note that once the problem of pleating is solved more importance is placed on the effectiveness of the ultrasonics, because this is the only factor that limits the achievable reduction. It is for this reason that the research into ultrasonic dies has been carried out at CarnaudMetalbox.
1.1.5 Methods of generating high power ultrasonic vibrations

To generate and maintain vibrations in an object a transducer is normally used to convert an electrical signal to mechanical motion. A simplified description of the two most common types of transducer (magneto-strictive and piezo-electric - see figure 1.11) will be given here. The published literature relating to ultrasonic transducers for power generation is discussed in section 1.3.3.

The first type of transducer works on the principle of magnetostriction, by which a magnetic field causes elongation of certain metals (nickel alloys are usually used). A coil of wire wrapped around a metal core creates a magnetic field proportional to the electric current and the core expands and contracts as the field changes. A biasing field is required to obtain a (relatively) linear response; this may be achieved using permanent magnets or a dc biasing current. The core is usually laminated to minimise eddy currents within it.
1. Introduction

The piezo-electric transducer consists of a number (normally 2 or 4) of piezo-ceramic disks clamped between metal blocks by a high tensile screw. Under the action of an applied alternating voltage the disks expand and contract, transmitting vibrations to the blocks. The disks are liable to fracture if subjected to tensile stress so they are preloaded by the blocks and screw to ensure that a compressive stress is maintained at all times.

Magneto-strictive transducers are extremely robust (almost unbreakable) and can work over a wide frequency band, so are tolerant of frequency mismatching; their disadvantages are largely associated with inefficiencies caused by non-linearity in the magneto-strictive effect, eddy currents in the core and resistance losses in the wire coils. These limit the vibration amplitude that can be generated and can cause overheating (the coils are often wound with PTFE-insulated wire to prevent melting). Also the laminated cores are not easily fitted to a vibrating tool - often the cores are brazed to a solid part on which the tool can be fitted using a stud. The addition of this part naturally increases the losses still further. The recent development of magnetostrictive transducers based on rare earth - iron alloys may lead to solutions to these problems (Lhermet & Claeyssen [9]).

The piezo-electric effect tends to be more linear, and because it converts the electrical signal directly to mechanical movement there are no other sources of inefficiency. For this reason they operate with much higher efficiency and (given that only a certain amount of power is available) they can maintain a higher operating amplitude. Some early transducers of this type had a poor reputation for reliability, with fracture of the bolt and/or the disks as the mode of failure. This was to some extent a by-product of their high efficiency. Given a fairly low level of power input with the transducer under no load a very high vibration amplitude could be generated - causing tensile stresses high enough to destroy the transducer. The solution to this problem was in developing intelligent electrical power supplies capable of varying the power input to maintain a constant (safe) amplitude in the transducer.
Most of the work that has been carried out in the field of metal forming has used magnetostrictive transducers, largely because most of the research was carried out in the 1960's and 1970's when the piezo-electric type was not fully developed. More recently in the plastic welding industry the piezo-electric transducer has become the accepted standard. Several manufacturers of equipment for this industry have now developed highly efficient transducers that are neat, self-contained and have a high power capacity. For this research the piezo-electric transducers were chosen in order to achieve maximum amplitude under load. They have proved extremely efficient and reliable in this application. A magneto-strictive system has also been used for some research work where its lack of efficiency is not important and its relatively wide frequency range is an advantage (for vibrating the die at non-resonant frequencies).
1.2 ULTRASONIC SYSTEM FOR FORMING AEROSOL CANS

The aim of this section is to discuss the requirements of an ultrasonic system for forming steel aerosol cans. First the possible mechanisms of reduced forming force are described, and the most likely ones described in detail. This effectively fixes the type of ultrasonic die. Other requirements of the vibrating system are then discussed.

1.2.1 Proposed mechanisms of force reduction from earlier work

The reasons proposed for the observed reductions in forming force generally fall into one of three categories: changes to the material properties of the metal being formed, changes to the stress state and changes to the frictional state. A brief discussion is included here while the published work is reviewed in detail in section 1.3.1.

Some researchers, notably Langenecker [13], [14] and Izumi et al. [21] have attributed an apparent softening of the material to preferential absorption of ultrasonic energy at the grain boundaries, leading to localised heating and facilitating flow.

Another mechanism, proposed by many researchers, eg. Nevill and Brotzen [11], Pohlman and Lehfeldt [16] and Winsper and Sansome [28], [29] is that the reductions in forming force can be attributed to changes in the stress state during the vibration cycle. The argument can be applied to axial or radial excitation of the die. For axial vibration the extra stress generated by a "push" each cycle is superimposed on the average stress level. Provided the total stress exceeds the yield strength of the material then yielding will take place once per cycle, while the average stress is less than the yield strength of the material. In many metal forming applications (including this study) the force which can be applied is limited by the strength of the component. Superposition of axial stress will not help this type of process because the vibration stress generated will be applied equally to the component, causing it to yield at an average stress level lower than its actual strength. The application of radial vibrations, however, is potentially more useful. Here the
1. Introduction

Vibrations may cause an increase in the hoop stress in the forming region (assisting its yielding in this direction) without a corresponding increase in the axial stress that the component must withstand. The process of stress superposition by radial squeezing is known as swaging. This was demonstrated in a laboratory experiment by McQueen and Sansome [44].

One further complication here is that when the forming force is reduced (by whatever means) the material is, by definition, less stressed, so less work hardening would be expected. Reduced work hardening would also lead to an apparent softening of the material with ultrasonics, although the effect of this would be slight.

The final proposed reason for force reduction is a change to the frictional conditions. Friction changes have been measured in laboratory tests, notably by Pohlman and Lehfeldt [16] and by Polanski et al. [17]. The friction coefficient may be reduced by separation of the surfaces, reduction of the normal force or "pumping" of lubricant over the surface. Alternatively the direction of friction may be changed - it may work at an angle to the direction of motion or, ideally, may be reversed so that friction assists the process. There is a theoretical basis for all of these possible changes:

Vibrations normal to the surface will cause a variation in the normal force. During that part of the cycle when the normal force is reduced sliding can take place with reduced friction. At the opposite part of the cycle, when the normal force is increased, the sliding may stop. Thus sliding takes place only when the normal force is reduced, so the friction force is correspondingly reduced. If the variation in the normal force is such that zero force is obtained then the surfaces separate for a part of each cycle, and sliding can take place with zero friction during this period. The reduced normal force also contributes to the lubricant pumping effect.

Vibrations parallel to the surface can cause changes in the direction of friction. If the direction is parallel to the surface but perpendicular to the direction of motion then the angle of the friction vector will be altered. The
1. Introduction

magnitude does not change (using the Coulomb friction model) so the component of friction in the direction of motion is reduced. If the vibrations are induced parallel to the direction of motion, and the vibration velocity is greater than the sliding velocity, then there will be a part of the cycle during which the direction of sliding is reversed. This must result in reversal of friction during that period. Note that these changes are heavily dependent on the relative sliding velocity - at higher forming speed the effect of the vibrations is reduced.

1.2.2 'Most likely' mechanisms in this application

From the available information it was thought that vibrations normal to the die surface (i.e. radially) would give the most beneficial effect, and this was largely confirmed by some early trials comparing the effects of axial and radial excitation. Therefore the design and optimisation work has been focused on radial-mode dies. The "most likely" force reduction mechanisms for radial vibrations are swaging (gradual deformation by successive radial squeezes) and friction reduction (figure 1.12).

![Diagram of swaging and friction reduction](image-url)
The swaging process has the potential (ideally) to reduce the forming force to zero, but this is dependent on many factors including particularly the forming speed. The faster the can is advanced into the die the more deformation will be required during each cycle, and so the higher the forming force will be.

In contrast friction reduction does not have the potential to reduce the forming force beyond a certain limit. When friction is eliminated a forming force will still be required because work must be done to deform the material of the can. The forming speed should not affect the forming force because the friction reduction depends only on the normal force, not on the sliding speed.

Thus the character of these two mechanisms is quite different, although their apparent effect on the process (a reduction in the forming force) may be similar. In fact it is very difficult to definitively state which mechanism describes the real situation, but the measurements presented in section 6.5 correspond closely to the friction reduction model in two ways. Firstly the forming force did not increase at higher speed (if anything it was found to be reduced), and secondly the forming force with ultrasonics corresponded very closely to the force calculated in an analysis that assumed zero friction (appendix 1).

### 1.2.3 Ultrasonic system for forming aerosol cans

It follows from the proposed mechanisms of force reduction that vibrations perpendicular to the can surface are required. This implies that the die must vibrate in a radial mode - in which it alternately expands and contracts with every point on the inside surface moving radially with the same amplitude and phase. Accordingly the aim of this work has been to develop such a die, building on past work (as discussed in the literature review) but improving the efficiency of the vibrating system to obtain maximum amplitude while minimising operating costs.
For this reason the piezoelectric type transducer was chosen, and proprietary units from two manufacturers of plastic welding equipment have been used. Only one of these transducers can normally be fitted to a die (in contrast to the magnetostrictive type) but provided that the resonance characteristics of the die are correct this is not a problem - a single transducer fitted at one point on the die can excite the required uniform radial mode. The first priority in the design of the ultrasonic system, therefore, is to ensure that the resonance characteristics are correct - this is discussed in detail in chapters 3 and 4.

The second problem for a radial-mode vibrating system is that no part of the die is stationary - there are no nodal points. A mounting system is required which permits the die to vibrate freely while providing a stationary point by which the whole system can be fixed to the machine. Again there is some prior art but a more efficient, simple and robust system has been designed specifically for this application (described in chapter 5).
A typical system used for ultrasonic necking of metal cans in this project, comprising piezoelectric transducer, radial-mode die and tubular mounting system is shown in figure 1.13.
1.3 LITERATURE REVIEW

The purpose of this section is to identify and review previous work that may be relevant to the present study. Published papers dealing with the use of high power ultrasound in metal forming processes are listed and discussed to indicate the history of this technique. Other industrial applications, methods of analysing die vibrations and materials for ultrasonic tools are also reviewed.

1.3.1 Application of high power ultrasound to metal forming.

Much of the early research work was largely theoretical and concentrated on apparent changes to the material properties under the action of the ultrasonics. Blaha and Langenecker [10] strain tested monocrystalline metal samples immersed in tetrachloromethane irradiated with ultrasound over a wide frequency range (up to 800 kHz). The ultrasound led to increased strain to fracture and increased ultimate tensile strength. Finding that ultrasonic energy was more effective than thermal energy at reducing the tensile stress, they suggested that ultrasound might be more readily absorbed at the dislocation sites that gave rise to plastic flow.

Other work (e.g. Nevill and Brotzen [11]) suggested that a reduction in flow stress could be attributed simply to superposition, because although the mean applied stress was reduced, the peak stress was equal to the static yield stress. Another alternative, proposed by Severdenko and Klubovich [12] was that ultrasound was simply analogous to increased temperature, causing a reduction in the rate of work hardening.

Langenecker [13], [14] showed that in tensile tests the results could not be explained by acoustic stress alone. He subsequently [15] discussed the various possible mechanisms and suggested that at low energy density the superposition mechanism operated, while at high energy overheating of the sample caused softening.
Besides these "volume effects" ultrasound has also been reported to affect frictional characteristics (the "surface effect"). For example Pohlman and Lehfeldt [16] investigated not only "internal friction" effects (stress superposition) but also "external friction". Applying vibrations in three orthogonal directions to a sliding contact they found that in all cases friction was reduced. This was attributed to the vibrations shearing of the welded asperity junctions, which would permit free sliding. Later Polanski et al [17] also measured changes in friction coefficient using a wedge test and found a 40% reduction in dry friction and a 20% reduction under lubricated conditions.

Given these promising results ultrasonic vibrations were soon applied to industrial metal-forming processes, particularly forging, extrusion and drawing. Ultrasonic vibrations were used in addition to the normal equipment for the processes concerned and the aim was to improve forming conditions.

In early work on forging aluminium with ultrasound, Severdenko and Klubovich [12] reported dramatic results, including reduction of forging force to zero, virtual elimination of "barrelling" and reversal of the residual stress distribution. It was suggested that these benefits were a result of reduced friction, elasto-plastic wave formation and thermal softening effects. In similar experiments Balamuth [18] and Kristoffy [19] in the USA and Izumi et al [20], [21] in Japan observed force reductions which at low vibration amplitude could be accounted for by superposition, but at higher amplitudes the effect was greater than would be expected for superposition alone. In these experiments, however, the temperature of the test piece rose by as much as 300 °C, and the force reduction was attributed to a combination of stress superposition and thermal softening. The altered residual stresses and reduction in "barrelling" reported by Severdenko were not observed by Izumi.

In the process of extrusion of metals with applied ultrasonics Tursunov [22] measured force reductions of 20 to 40%, and noted a reduction in microhardness of the product. He attributed the effect to a reduction in yield
stress caused by the ultrasonics. In the USA Jones [23] (of Aeroprojects Inc.) extruded lead and aluminium with applied ultrasonics and found that by vibrating the die (the most effective option) the force could be reduced by 15%, or the rate of extrusion (at constant force) could be increased by 88%. In this case the improvement was attributed to friction reduction. Tarpley [24] reported even greater improvements: force reduction of between 15 and 30% or rate of extrusion increased 100 to 300%.

In wire drawing ultrasonic vibrations have generally been applied to the die along the wire axis. Early investigations by Severdenko and Klubovich [25], Robinson [26] and Boyd and Maropis [27] for a wide range of materials and draw speeds showed a significant reduction (up to 65%) in the draw load and a slight reduction in the microhardness of the drawn wire. However using ultrasonics also led to severe pick-up in the die and a poor surface finish. By contrast Jones [23], while noting a similar reduction in draw force, found that the material properties of the drawn wire were unaffected and the surface finish was improved. In this case the improvements were attributed to a reduction in friction. By measuring the instantaneous stress in the wire (as opposed to the mean stress), Pohiman and Lehfeldt [16] and Winsper and Sansome [28] determined that the peak stress was unchanged. This indicated that the drop in apparent (mean) stress was due to superposition of the acoustic stress.

In a forming process limited by the strength of the product this effect would not be useful because the apparent strength of the wire would be reduced in proportion to the apparent drop in forming force. Nevertheless Winsper and Sansome [29], [30], using three dies with the centre one vibrating achieved a "genuine reduction" of up to 50 lbf in the draw force (total force figures are not given so the percentage improvement is not known). Another way to avoid this problem would be to use radial vibrations of the die and take advantage of a swaging effect due to the workpiece being compressed laterally. There are few reports of work in this area probably because of the
difficult of producing small aperture radial-mode ultrasonic dies, but a similar
effect can be obtained by using a split die and vibrating one half laterally, or
by installing the die in the centre of an axial mode resonator. Using this
technique Oelschlagel and Weiss [31] observed force reductions from 37% to
62%. Later work by Lehfeldt [32] also included investigation of this type of
die.

In the drawing of tube, where the die aperture is often larger, there was more
scope for using radial vibrations. In early work, however, the axial mode was
used as for wire drawing. Mainwaring [33] and Jones [23] tested axial-mode
dies in plug drawing and reduced the draw force by up to 80%. Surface finish
and hardness of the product were found to be unaffected. In this process
(which uses a fixed plug inside the tube) it is also possible to axially vibrate
the plug to obtain similar results. This is the process that Aeroprojects put
into production claiming many advantages including reduced draw force,
improved surface finish on the bore and the ability to achieve greater
reductions and produce more complex sections. Again Aeroprojects attributed
these benefits to a reduction in friction. Boyd and Kartluke [34], also derived
a formula for the effects of ultrasonics based on this theory.

A notable researcher in the UK at this time (1960's and 1970's) was Prof. D.
H. Sansome along with his team at Aston University, Birmingham. Some of
their research into wire drawing is described above. Their research into tube
drawing began by using axial vibration of the plug (as Aeroprojects) -
Winsper and Sansome [35]. The beneficial effects (reduced draw load and
improved surface finish) were attributed to a combination of stress
superposition and friction reduction. In later work Kariyawasam, Young and
Sansome [36] used radial die vibrations with a plug bar of tuned length (so
that axial vibrations would be induced in the plug) to achieve a force
reduction ranging from 10 to 30%.
This team also applied radial ultrasonic vibrations to deep drawing and wall ironing processes. Biddell and Sansome [37] used a variety of radial-mode dies to achieve improved depths of draw, especially at low speed, and concluded that the effect was a result of reversal of the friction vector. Later Biddell [38] reported further work again using radial dies which showed an increase of draw depth of about 12%, but this was not always achieved. The work also extended to wall ironing but it was found that for high reductions the ultrasonic system was unable to maintain the vibration amplitude. Tisza [39], [40] also reported work on ultrasonic deep drawing, in this case with a complex combined axial-radial mode die which converted axial vibrations at the transducer to radial at the die. This overcame the perceived need to use a large number of transducers arranged around the die to generate a uniform amplitude. Using a single magnetostrictive transducer an amplitude of 15 μ was achieved and the achievable deformation was increased by up to 15%. The effect was attributed to reduced "friction factor", friction vector reversal and a modified tensile strength under the action of the ultrasonics.

To avoid problems maintaining amplitude and to approach the research in a more fundamental way Sansome's team developed simpler test processes - the wedge draw test simulated deep drawing while deforming only a sector of a cylinder, while the strip ironing test simulated wall-ironing in a flat sample. Initial results for the wedge test were mixed. Smith, Young and Sansome [41], [42] reported that vibrating the punch alone produced an apparent reduction in the forming force (due to stress superposition) but no significant benefit in achievable depth of draw. However by radially vibrating the die an increase in draw depth of the order 5% was achieved. Best results were obtained by radially vibrating the blankholder [43] which gave an increase in depth of draw of the order 10%.

The wall-ironing process was simulated by drawing a strip of metal through a pair of flat dies. McQueen and Sansome [44] vibrated both dies in a
perpendicular direction at approximately the same frequency (within 50 Hz). The maximum area reduction achievable was increased from 63 to 80% with ultrasonics. Note that this process simulates wall ironing with a radial-mode die and a radial-mode punch. Vibrating one die only would perhaps have been more realistic - simulating the wall-ironing process with a radial-mode die only.

The application most similar to the work described in this thesis is "tube sinking" (a term used in the tube drawing industry that is equivalent to "necking" in the can industry). In this process one end of a tube is forced into a die to reduce its diameter. Aeroprojects' work on tube drawing was also applied to tube sinking in the early stages [23]. Westinghouse also used an ultrasonic process for tube expansion [45].

More recently in eastern Europe highly relevant work was reported by Skachko, Pashchenko et al [7], [8]. This group applied high-power ultrasonics to the forming of an aerosol neck, using ultrasonic dies in the conventional process for necking aluminium aerosols, i.e. multi-stage necking with a simple parallel plug. He found that it was possible to reduce the number of operations required from 9 to 3 using ultrasonic vibrations in the axial and / or the radial directions. It is important to remember, however, that this work was applied to aluminium aerosols. Using harder, thinner material (steel) the limiting factor on reduction achievable in a conventional necking operation is buckling in the circumferential direction (pleating), and this is not significantly affected by vibrations.

In almost all applications a significant improvement in the forming process was reported when using ultrasonics, either a reduction in the forming force or an increase in the apparent formability of the workpiece. Various theories have been proposed for these observations, as further described in section 1.3.1. Despite this, widespread application in industry did not follow. Several review articles suggested that full production status was imminent [46], [47], [48], [49] and in another Biddell and Sansome suggested that industry was to
blame for not applying the technology [50]. Industry would probably argue that the benefits were insufficient to justify the extra cost and complication of the ultrasonic equipment. High-power ultrasonics have been fully accepted in another section of industry (plastic welding) as described in section 1.3.2.

For a more detailed review of early work on vibratory forming (both low and high frequencies) see Dawson et al [51]. For a review of work at Aston University see Sansome [52]. Jones' review of work at Aeroprojects [23] is particularly comprehensive and includes a table of experimental results obtained from many industrial trials.

1.3.2 Other industrial applications of high power ultrasonics

Other applications of high-power ultrasonics in industry include cleaning, plastic welding, metal welding, cutting. These have largely developed in a different section of industry to the metal-forming applications, and the development of equipment to generate ultrasonic vibrations has taken a different route (this will be further discussed in section 1.3.3). Few reports of the developments in these areas have been published, possibly because most of the work has been undertaken by industry rather than academia. Nevertheless these applications will be discussed, with reference to sales literature where necessary.

Ultrasonic cleaning involves immersing the workpiece in a fluid irradiated by ultrasound, often from several sources (to obtain a relatively uniform field). The cleaning action is enhanced by agitation of the fluid, which helps it to penetrate any awkward cavities in the workpiece, and by cavitation. This is the catastrophic collapse of vapour bubbles that produce "micro-shocks" capable of dislodging dirt and even eroding the component surface - the standard test for intensity involves immersing a piece of metal foil for a set time and then counting the holes! Traditionally the fluid was agitated by multiple piezo-ceramic disks arranged around the walls of a tank (see, for
example Perkins [4] but more recently the disks have been replaced by one or more tuned probes transmitting the vibrations into the fluid from above the surface. One such system is produced by Telsonic and described in OEM Design [53]. The probe systems are claimed to be more efficient because less energy is lost to the tank walls.

Another application using broadly similar equipment is sonochemistry - the use of ultrasound in liquids to affect the rate and/or products of a chemical reaction. For further details see the works of Lorimer and Mason, for example [54]. Ultrasound has also been applied to biological samples, offering improved mass transfer between cells or (at higher levels) disruption of cell walls - see Sinistra [55]. For a detailed description of the ultrasonic equipment and instrumentation necessary for this type of work (sonochemistry and biological effects) see Perkins [56].

A similar application is ultrasonic deburring [57]. Standard cleaning equipment is used with an acid slurry (chemically and mechanically aggressive) to remove unwanted material from a machined surface. The use of ultrasonics is claimed to provide better control over the rate of removal because the ultrasonic intensity can be conveniently adjusted.

Previous applications of ultrasonics for machining have been described by Neppiras [58] and Markov [59] (edited by Neppiras). The latter is particularly detailed, describing techniques for ultrasonic cutting, drilling and grinding. Typically these operations use an abrasive slurry containing hard, sharp particles, along with a relatively soft tool, into which the hard particles become embedded. The machining is accomplished by breaking microscopic particles from the workpiece.

Welding of plastics using ultrasonics has become a standard technique in many industries including packaging (e.g. joining card/plastic laminates for carton manufacture), automotive (e.g. light clusters, bumpers, fuel tanks) and many others. The process involves vibrating one component against the other while clamping them tightly together, and is described in great detail in
booklets produced by the German Electrical Manufacturers Association (ZVEI) [60] and the equipment manufacturer Herfurth (edited by Rische and Abel [61]). As one component slides against the other the sliding generates frictional heat that softens both surfaces, allowing them to merge under the action of the clamping force. Typically the direction of vibrations is perpendicular to the joining faces, although this may also generate sliding motion parallel to the surface. It is important to maintain the clamping force for a short time after the ultrasonics have been turned off to allow the weld to cool and harden. The advantages of this process are efficiency (because the heating is localised at the weld area) and convenience, indeed the weld may be some distance away from the ultrasonic tool, although resonance of the components themselves may limit the weld integrity (Jagota and Dawson [62]).

Metals can also be welded using vibration energy, see Devine [63]. Ultrasonic equipment differs from that used for plastic welding in that the vibrations are applied parallel to the surfaces to be welded. The advantages claimed for this technique are its ability to weld dissimilar metals and to weld through oxide layers without the use of corrosive flux. It has been used for welding of small components, e.g. connectors to car battery leads, but is generally limited to relatively thin sections. An exception is the work described by Tsujino et al. [64] which showed that aluminium plates up to 10 mm thick could be successfully butt-welded.

More recently Tsujino et al [65] have also reported an ultrasonic sintering process that improves the density and uniformity of the material. This used three independent ultrasonic systems (a radial-mode die with axial-mode plugs at each end).

A very different application, operating currently at much lower power, is in ultrasonic motors. For example in robotics, Schoenwald et al. [66] describe an ultrasonic gripper system. This is an example of a simple rotor system - driving a contact at an oblique angle to the surface so that a small
incremental movement is generated on each vibration cycle. More complex (but potentially more efficient) systems produce drive by generating an elliptical motion at the surface. This can be achieved by generating a travelling wave in one component, which may be generated by superposition of two (resonant) standing waves at a fixed phase angle. An example of this type of work is LeLetty et al. [67], who described rotary ultrasonic motors using resonant rings operating in high harmonic modes similar to the unwanted modes observed in this project. Furthermore the motor design was achieved using finite element analysis (ATILA) to include the piezo-electric effect in three dimensions. The generation of elliptical surface motion, if it could be achieved at higher power and for the more complex geometry of a metal-forming die, could offer a major improvement to the forming process. This is discussed further in section 7.2.3.

Another totally different application is sonar - the use of reflected sound waves in liquid or air to detect distant objects. Note that this is in principle quite similar to non-destructive testing, but on a much larger scale. Because of the large scale the power used in sonar detection is high, and generation techniques are broadly similar to those used for other high-power applications. The most important parameter in this case, however, is generally the coupling to the transmission medium. See, for example, Gough and Knight [68] for a discussion of the requirements of this application. This paper is notable also for the use of admittance circle plots to indicate the performance of the ultrasonic system - see section 6.2.2.

Another use of high-power ultrasonics is in material testing, where the relatively high stresses combined with the high frequency operation significantly shorten the time required to characterise the material. For example Soderberg et al. [69] investigated fretting wear while Puskar [70] and Chapman [126] among many others, studied fatigue testing under the action of ultrasonics. Note the danger this highlights - all ultrasonic tooling is prone
to fatigue failure, and careful design and control of operating conditions is necessary to prevent this. This is discussed in section 3.7.

1.3.3 Methods of generating high power ultrasonic vibrations

Transducer materials to convert electrical signals to mechanical motion (or vice versa) have been developed since the 1920's. Neppiras' review [71] describes the history and characteristics of many transducer materials, including nickel / cobalt alloys (used for most high-power magnetostrictive transducers) and lead-zirconate-titanates (used for high power piezoelectric transducers). For a more recent review of the piezoelectric / electrostrictive materials see Uchino [72], who has studied actuator applications (deformable mirrors, dot-matrix printers) and ultrasonic motors.

Whymark [73] gives a detailed description of the design and evaluation of magnetostrictive transducers, including consideration of performance under load. At this time - 1956 - the optimum efficiency of this type of transducer was particularly low (42%).

Perkins [4], while reviewing all aspects of high-power ultrasonics, compares the two types of transducer. He quotes potential efficiencies of 55% for magnetostrictive and 90 to 95% for piezoelectric.

Shelley [74], reviewing the work of Sansome's team at Aston, suggested that piezo-electric transducers had been found to be too delicate for industrial applications. Rees and Rippon [75] also found piezo-electric transducers unreliable but noted in contrast the low efficiency (65%) of magneto-strictive systems. Much of the unreliability observed was probably the fault of the control systems. Wearden [76] describes a control system for tube drawing (with magneto-strictive transducers) which automatically adjusts the drive frequency to maintain resonance within 0.3%, or 60 Hz at 20 kHz. For plastic-welding systems using piezo-electric transducers a much more precise frequency control is required (for example the Telsonic system [77]
maintains resonance within less than 10 Hz). Furthermore Wearden does not mention amplitude control at all, but control of the transducer amplitude is vital to the correct operation of a piezo-electric system because under no-load very little power is required to develop high amplitude (and hence high stress) in the transducer and tooling. Sonkin [78] also discusses the design of ultrasonic generators.

For further information the German Electrical Manufacturers Association (ZVEI) has produced a useful manual of the application of ultrasonics to plastic welding [60] which includes details of the transducers and control systems used. The equipment manufacturer Herfurth has also produced a booklet of similar content based on their own work [61]. This is one of several manufacturers of equipment for this industry who have now developed highly efficient transducers that are neat, self-contained and have a high power capacity. Sophisticated instrumentation and control systems have also been developed to operate them (see also [53]).

1.3.4 Analysis of vibration characteristics

The design of axial-mode ultrasonic tools is relatively straightforward provided the diameter of the tool is less than about a quarter wavelength. Selecting a length which is an integer number of half-wavelengths ensures that the tool (sonotrode) will be tuned to approximately the required frequency, and fine tuning can be achieved by further machining of the length. Young, Winsper and Sansome [79] even suggested that a tool could be attached to the end of an axial-mode resonator by simply removing an equal mass of material from the free end. This practice is not generally recommended because the joint will not be at a stress node and the fixing is likely to be destroyed by any significant vibration amplitude. The design of radial resonators is much more difficult, firstly because the relationship between dimensions and resonant frequency is more complex and secondly
because the harmonic natural frequencies do not appear in a regular series (see section 3.2).

Analysis of the vibrations of rings specifically for the ultrasonic die application was carried out by Biddell and Sansome [37], [38] and by Young, Winsper & Sansome [80], [81]. These analyses are limited to specific die shapes (flat, tapered, exponential, etc.) and to the radial axisymmetric mode only. The analysis of appendix 2 is similar but anticipates analysing the die as a series of thin rings. This permits analysis of any die shape and a number of different material properties (e.g. for dies with a hard insert of a different material). Like the earlier work however this is limited to the axisymmetric radial mode of vibration, and other modes (as described in section 3.1) can be very important. Analysis of non-axisymmetric modes of rings is discussed by Den Hartog [82] and in more detail by Timoshenko [83] but this work only covers rings of thin cross-section that do not accurately model typical ultrasonic dies. Blevins [84] gives empirical formulae for natural frequencies of rings, plates and tubes, including the harmonic modes, but the first harmonic is listed as a rigid body mode. The analysis of non-axisymmetric modes of thick rings was carried out using finite elements by Gladwell and Vijay [85] and as a full (classical) analysis by Hutchinson & El-Azhari [86], [87], although their equations required matrix arithmetic with large matrices and iterative solution. Both of these analyses still involved some simplification - the shape of the ring was limited to a hollow cylinder of uniform thickness, so this work has been used only to verify the finite element results (section 2.8).

Finite element analysis has been used for assisting in the design of ultrasonic tools notably by Derks [88] who was concerned with the design of plastic welding sonotrodes. This involved finite element analysis similar to that described here but with the aim of promoting axial modes of vibration in complex cylindrical and rectangular resonators. These are resonators for which the dimension perpendicular to the vibrations is greater than a quarter wavelength, so the simple approach to sonotrode design does not work. His
work would be extremely valuable in the plastic welding industry but is not
directly relevant to the design of radial-mode ultrasonic dies.

The finite element method is now a well-established tool for all kinds of
engineering analysis. With the ever increasing availability of computing
power and the continuing development of user-friendly interfaces this trend is
certain to continue.

1.3.5 Materials for ultrasonic tools

The materials that are believed to be most suitable for the ultrasonic tooling
in this application are described in detail in section 3.7. Two types have been
selected corresponding to the different requirements of the inner and outer
components of the die. For the hard, wear resistant inner die suitable
materials include tool steels, steel matrix cermets and ceramics, while for the
fatigue-resistant die outer high strength alloys of aluminium and titanium have
been chosen. For clarification of the die construction see figure 1.13.

For general information on tool steels see the Metals Reference Book [89] or
the Properties of the En steels [90]. To obtain a hard, wear-resistant surface
finish various heat treatments are possible. In particular for the nitriding steel
used in this work a plasma-nitriding treatment has been used to produce a
thin layer of hard material on the surface with minimal distortion of the part.
The harder surface also gives a lower coefficient of friction that assists the
forming process. Both the surface hardness and the friction coefficient can be
further improved by the addition of a titanium nitride coating (applied on top
of the plasma-nitrided surface to prevent cracking of the coating due to
collapse of the substrate). These surface treatments are described by Staines
[91]. The steel-based cermet used in this work is Ferro-titanit Nikro 292,
manufactured by Thyssen [92] (note that this particular grade has now been
discontinued by the manufacturers - the similar Nikro 128 may be used in its
place). The ceramic material that has been used is sialon, a modified silicon nitride described in general by Wilson [93], specifically Syalon 101 [94].

For further information on high strength aluminium alloys (and particularly the L168 aircraft grade used in this work) see the Aluminium Reference Book [95]. The general-purpose titanium alloy Ti-6Al-4V (i.e. 6% aluminium, 4% vanadium), which has also been used, is described in detail in [96]. For a more general discussion of the engineering applications of titanium alloys (although curiously ultrasonic tools are not mentioned) see Hanson [97].

One common requirement of both components of the ultrasonic die is a low rate of acoustic loss (i.e. energy loss caused by material damping). This is particularly important for the die outer since it is generally the larger component, and the total power loss from the system will depend on the amount of each material as well as their individual damping coefficients.

Unfortunately information on the damping characteristics of materials is rarely available. Even when data is available, test results must be treated with caution because the damping value obtained is dependent on the frequency and amplitude of vibration and on the nature of the test. This is described in the Metals Reference book [89] which includes a warning regarding the accuracy of the information because it is collected from various sources. A collection of results for different materials tested under the same conditions is more useful, for example Adams [98] gives good comparative data for several grades of steel, brasses, bronzes and aluminium alloys at 11.6 kHz (unfortunately titanium alloys are not included). This work suggested that the best materials for ultrasonic applications (i.e. those with the lowest damping) are aluminium alloys, followed by brasses and bronzes.

The selection of material for the die inner and outer components depends to a large extent on the requirements of the process (e.g. wear resistance and hardness of the inner according to the abraisiveness of the material to be formed), but further constraints are put on the material selection by the essential resonance characteristics of the assembly. These are discussed in
detail in section 3.4. One further group of materials has been considered because of the potential to provide more flexibility in this area: aluminium matrix composites. These materials (comprising ceramic particles in an aluminium matrix) offer a significant increase in Young's modulus with minimal change in density. This increases the sound velocity in the material so that for a given resonant frequency a larger component will be required. In producing ultrasonic dies for larger diameter forming operations this change could be crucial to producing a working ultrasonic die. Furthermore the change in modulus depends on the proportion of ceramic in the composite - so by selecting the appropriate mix the material properties could be customised to suit a particular application. The damping properties of these materials have been investigated by Bhagat et al [99], [100] who found slightly higher damping in the composites than in the base aluminium alloy, but the difference was small enough at high frequency not to cause problems for ultrasonic equipment, although it was suggested that at higher strains the composite should have higher damping. At present these materials are new to the market and very expensive, but as production levels increase the cost should fall and in future these materials may become common in the ultrasonics industry.

For any combination of materials used to construct an ultrasonic die the maximum safe amplitude at which the die can operate will generally be limited by fatigue of the die outer. One very effective method of improving the fatigue resistance is shot peening. This generates a compressive stress in the surface layer to resist the growth of microcracks that are always present there. The technique is described and analysed by, for example, Fuchs [101].
1.4 SAFETY OF HIGH POWER ULTRASONICS

With the introduction of any new technology to the industrial workplace any possible health risks must be carefully considered. In the case of high power ultrasonics, as discussed in section 1.1, other industrial applications exist, and it is possible to learn from this experience. In particular, ultrasonic equipment has become almost universal in the plastic welding industry, and ultrasonic cleaning is also a commonly used process.

The health risks of high power ultrasonics fall into two categories: risks from direct contact with the vibrating parts and risks from airborne noise (including audible noise and ultrasound). Besides these there may be other risks normally associated with moving machinery (e.g. trapping points).

Normal machinery risks can be avoided by suitable guarding, as described in BS5304:1988 [102]. In the case of the aerosol forming process the forces are high enough to present some risk of amputation of a finger or even a hand. Access to the machine is also required (for setting or clearing jams) so an interlocked guard system is required.

If a user comes into direct contact with a vibrating part there a risk of disruption of the tissues and / or rapid energy transfer, causing a burn (see, for example Hill [103] for a discussion of physical effects). Despite this, it is common (though not recommended) practice in the plastic welding industry to evaluate the performance of a sonotrode (ultrasonic tool) by the "feel" of the vibrations. This practice does not normally cause injury because the fingers are only lightly touched on the vibrating parts, so energy transfer is minimal. To prevent deliberate or accidental contact with a vibrating part the ultrasonics should be electrically interlocked to the guard system.

In addition to studying physiological effects of direct contact, other researchers, e.g. Acton [104] and Wiernicki & Karoli [105] have proposed standards for safe exposure to airborne ultrasound. These indicate that while the effects of airborne ultrasound are not fully researched, there appears to
be less risk from ultrasound at any given sound level than from audible noise at the same level. Applying normal legal standards for noise level / exposure time should therefore ensure that there is no risk of hearing damage. Airborne high-frequency sound is easily deflected by lightweight barriers so these standards are easily achievable.
1.5 SUMMARY

The following is a short description of the contents of the remaining chapters.

The analysis and design of ultrasonic dies rely heavily on finite element analysis. Chapter 2 describes the theoretical basis of this, plus practical considerations using the Ansys [106] finite element program. Some methods of simplifying the analysis are discussed, along with their advantages and drawbacks. Alternative analytical approaches are also considered in chapter 2, and several are used to verify the accuracy of the finite element results.

In chapter 3 the background information on finite element analysis is applied to the design of ultrasonic dies. To describe the various natural modes of the dies a new system of nomenclature was developed. Detailed information on the process of design and manufacture is given, and the major design considerations (materials, allowable stresses, geometric constraints, etc.) are also discussed.

Chapter 4 describes a simplified system that was developed to assist the initial design of new ultrasonic dies. Design information in the form of two dimensional contour plots is given for a range of materials in appendix 5. The use of the new system is demonstrated by means of examples.

In chapter 5 the problem of attaching a vibrating die to a fixed machine is addressed. There are no stationary (nodal) points on the die so a mounting system is required which will permit the die to move with the vibrations while at the same time locating it accurately. The shortcomings of existing systems are discussed and a new mounting system is introduced.

Chapter 6 describes methods of measuring the performance of the ultrasonic system (i.e. the die and its mounting, along with the transducer required to drive it). The measurements are aimed at both characterising the vibrations in use and evaluating the effect on the forming process. The results reproduced include summary tables of information for a number of different ultrasonic dies and graphs of the force variation during the forming stroke. The
accuracy of the finite element results is also indicated by comparison with the measured data.

In chapter 7 suggestions are made for areas of further work in this field. In particular, industrial pressures have required that producing an effective ultrasonic system has taken priority over understanding the reasons for its effectiveness. Alternative methods are discussed for studying the interaction between the vibrations and the forming process.

Finally chapter 8 lists conclusions drawn from the work described here, along with a discussion of their implications.
The design of ultrasonic dies depends very heavily on the use of Finite Element Analysis (FEA). Mathematical analysis based on classical theories has been used but the simplifications required to make the equations solvable (e.g. simplified die geometry or limitations on the vibration mode) ensure that the results are inadequate for complete die design. In any case the continuing reductions in the cost of computing power have made finite element programs so readily available that the complex analysis and equation solving approach would now be difficult to justify.

That finite element analysis is eminently suitable for analysing ultrasonic tools has been demonstrated by Derks [88]. His work on the design of plastic welding sonotrodes, and other relevant literature, is described in section 1.3.4.

Three-dimensional FE models have been used to analyse the ultrasonic dies but simpler means of analysis based on two-dimensional elements can also be suitable (as explained in section 2.4). This has made possible the use of sophisticated design optimisation techniques.

CarnaudMetalbox Packaging Technology has used the "Ansys" [106] finite element program since 1985. This is a general purpose program capable of structural, thermal, fluid, electrical and electromagnetic analysis using the finite element method.

This chapter describes how the program is used, the types of analysis and finite element models, and the accuracy and reliability of the results.
2. Finite Element Analysis

2.1 FUNDAMENTALS

For a fuller description of the finite element method the reader is referred to Bathe [107].

The finite element method relies on breaking down a large, complex object into a number of small, simple "building blocks". Each block is individually quite simple to analyse because it is only allowed to deflect in a limited number of ways. This does not lead to serious inaccuracies because the block can represent an arbitrarily small part of the object as a whole. In any sufficiently small part of the object the stress state can be considered uniform and so this small part is subject to simple deflections. Although the deflections of each block can be quite simple to calculate the analysis of a large number of interacting blocks requires the solution of a huge number of simultaneous equations. This is normally expressed as matrix arithmetic. The finite element program forms matrices to represent the object to be analysed and then solves the matrix equations which will produce the answers requested by the user.

In finite element terminology these blocks are called "elements". An element is defined by the positions of its corners and these positions are defined by points called "nodes". One node may be common to any number of elements grouped around it, and these elements will then be joined together (see figure 2.01).

The allowable motion of each node is defined by its "degrees of freedom". In the most general case a node will have 6 degrees of freedom - three translations and three rotations.
Although three dimensional elements like this are easiest to visualise for representing a three dimensional object in many cases there are other alternatives. Similar results can often be obtained using two-dimensional or even one-dimensional elements, but this will depend very much on the object to be modelled and the results required. The nodes joining these simpler elements naturally have fewer degrees of freedom. If the problem can be simplified in this way the benefit is greatly reduced computing power and time requirements and/or improved accuracy. The selection of element types is discussed in section 2.4.

The process of using a finite element analysis program at its simplest can be broken down into three stages:

2.1.1 Pre-processing

At this stage the user defines the object to be analysed (its geometry, material properties etc.), the influences acting upon it (in structural analysis
2. Finite Element Analysis

forces, pressures, deflections etc.) and the type of analysis to be used (static, dynamic, modal etc.). The user must also supply some information about how the analysis is to be done.

To define the object geometry a simple program might require the user to define the position of every individual node and the corner nodes of every individual element. Fortunately all major commercial programs now partially automate this process to assist the user. Ansys allows the user to define 3-D geometry by means of "volumes" of any convenient size within the object. The program will automatically generate all nodes and elements within each volume given some indication of the required element size. Other information is often required to fine-tune the analysis process for best results in the minimum time. An example of this is the use of master degrees of freedom in a reduced dynamic analysis (described in section 2.3).
To assist the user in defining the geometry and checking his work the pre-processing part of a finite element program is usually provided with extensive graphics facilities which can be used to produce pictures of the model as it is defined. This is vital because without it the slightest mistake could produce a model which completely failed to represent the real object and would give erroneous results. Figure 2.02 shows typical graphical output from the Ansys pre-processor. Another useful facility provided by some programs (including Ansys) is parametric model definition which allows any feature of the model to be defined in terms of a named parameter which can be conveniently modified. This facility has been used extensively as described in section 2.6 and section 4.4.

### 2.1.2 Solution

After the user has supplied all the necessary information the program can go ahead with the analysis. This stage may require a great deal of computer processing time and so is often done overnight or in batch-mode (i.e. the computer works on the problem only when it is not busy dealing directly with a user). First the model information is converted into matrix form. In structural analysis an object will be represented by its mass and stiffness matrices which reflect the distribution of mass and stiffness among the degrees of freedom of the model. Forces and deflections are converted to vector form. A matrix equation is formed and then solved for the analysis specified by the user. Typical matrix equations for the different analysis types are described in sections 2.2.1 to 2.2.3 The result may represent a set of deflections and stresses, natural frequencies and modeshapes or other results relevant to the analysis. At this stage the results are stored in matrix form and are not directly readable by the user.
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2.1.3 Post-processing

After solution the required results will be stored in file of numbers. The post-processing part of a finite element program is provided to help the user extract the particular information he requires and to ensure that other important information is not missed. As in pre-processing graphical output is an essential part of this process. Typically displacement is displayed using a distorted picture of the model on which the distortion is grossly exaggerated (numerical data is also provided to indicate the actual distortion). Stress is usually shown using contour plots with different colours representing different stress bands. The program can calculate components in three orthogonal directions or principal and Von Mises equivalent stresses for output in this way. For analyses which produce results varying with time or frequency (e.g. resonance curves produced by dynamic analysis where material damping is simulated), these can be displayed as graphs.

Examples of these plots are shown in figures 2.3 - 2.5 and also in other chapters.

Parametric output of results can also be obtained from the post-processor. This is essential to the optimisation process as described in chapters 3, 4 and 5.
2. Finite Element Analysis

FIGURE 2.03 ANSYS POST1 GRAPHICAL OUTPUT - DISPLACED PLOT

FIGURE 2.04 ANSYS POST1 GRAPHICAL OUTPUT - STRESS PLOT
2. Finite Element Analysis

![Graphical output](image)

Figure 2.05 ANSYS POST26 Graphical Output - Resonance Plot
2.2 ANALYSIS TYPES

Three types of analysis have been used in the design of ultrasonic dies and mountings: Static, Modal (mode-frequency) and Harmonic Response (forced dynamic). The following sections describe the function of each type of analysis and typical applications.

2.2.1 Static analysis

Static analysis calculates the stresses and displacements generated in an object by the action of forces (internal or external), pressures, thermal expansion etc. In the design of ultrasonic dies it has been used to determine the optimum interference fit which should be used to hold the die insert in the outer die (see figure 2.06). The reason for using two-piece dies is described in Section 3.7.
This is an unusual application for static analysis because no external forces are applied (dynamic vibration forces are considered later). The interference fit is simulated using "Constraint equations" which force a pair of nodes to separate by the required amount.

Static analysis would usually be used to solve equations of the form:

\[ K \cdot u = f_{\text{app}} + f_{\text{el}} \]

where

- \( K \) = Stiffness matrix
- \( u \) = Displacement vector
- \( f_{\text{app}} \) = Applied nodal force vector
- \( f_{\text{el}} \) = Element elastic load vector

(Note: Matrices will be shown here by capital letters in bold type and vectors by lower case letters in bold type.)

The applied force vector \( f_{\text{app}} \) will include all external forces and the internal forces applied by the constraint equations. In this case there will be no external forces and the internal forces will be equal and opposite forces applied to each pair of nodes linked by a constraint equation.

Solution methods will not be discussed here (for information see the Ansys Theoretical Manual (Kohnke [108])) but the result is the derivation of \( u \) the displacement vector which defines the displacement of each node and \( f_{\text{el}} \) the element force vector from which the nodal forces and hence stresses are derived.

Hence the result of a static analysis is a deflected shape and the stress state of the material. The deflected shape is important if the inside diameter of the die is tightly tolerated - the insert can be machined to a diameter slightly larger than the required value to allow for the slight shrinkage when the insert is fitted. The stress state is important in determining how well the die performs and its lifetime. This must be considered in conjunction with the stress state caused by the vibrations.
2. Finite Element Analysis

2.2.2 Modal Analysis

Modal analysis calculates the natural frequencies and modeshapes of an object. By definition this technique will consider only the mass and stiffness of the object to arrive at theoretical natural frequencies and modeshapes. Damping (which is present in all real systems) is ignored. Therefore the results of this type of analysis must be treated with some caution.

The effect of damping on a real system is to change the resonant frequencies and modify the modeshapes. When two modes appear at similar frequencies in the presence of damping the observed modeshapes will appear as combinations of the two natural modes, with the degree of distortion of each mode dependent on the amount of damping present. The effect on the resonant frequencies is to increase the separation between the modes. If there is a large frequency separation between a pair of vibration modes then there will be no significant effect on the modeshapes, and the resonant frequency will be lowered slightly by the effect of the damping. The materials from which the ultrasonic dies are made have been chosen for their low damping properties (Section 3.7) and so damping does not in this case significantly change the resonant frequencies. Also the separation between resonant frequencies in the area of interest must be fairly large (1-2 kHz) for the die to function correctly (Section 3.3). Therefore damping does not have a significant effect on the results in this case, and the modal analysis technique is extremely useful.

Modal analysis solves equations of the form:

\[( K - \omega_i^2 M ) \phi_i = 0 \]

where

- \( K \) = Stiffness matrix
- \( \omega_i \) = Circular natural frequency of mode \( i \)
- \( M \) = Mass matrix
- \( \phi_i \) = Mode-shape vector of mode \( i \)
2. Finite Element Analysis

Solution involves calculating the eigenvalues \( \omega \) and eigenvectors \( \phi \) for this equation. The user may select the range of \( i \) or specify a frequency range in which the eigenvectors and eigenvalues are to be calculated.

Hence the results of modal analysis are a set of natural frequencies and their associated modeshapes (the distorted shape of the object at one extreme of its movement). There is no information about the magnitude of the distortion so an arbitrary value is normally assigned and stresses caused by vibration are not usually calculated. However it is possible for the user to specify an amplitude value to which the modeshapes (where possible) will be scaled. In this case the stress state can also be calculated for each modeshape. The stresses must be considered in conjunction with the static stress caused by the interference fit to ensure that the die does not separate or fatigue in service (Section 3.8).

2.2.3 Harmonic analysis

Harmonic (or forced dynamic) analysis differs from modal analysis in that it does take into account driving force(s) and damping. It is a steady state analysis - assuming a constant sinusoidal response to one or more sinusoidal forcing function(s).

The equation to be solved is of the form:

\[
M \ddot{u} + C \dot{u} + K u = f
\]

where

- \( M \) = Mass matrix
- \( C \) = Damping matrix
- \( K \) = Stiffness matrix
- \( \ddot{u} \) = Acceleration vector \( \frac{d^2u}{dt^2} \)
- \( \dot{u} \) = Velocity vector \( \frac{du}{dt} \)
- \( u \) = Displacement vector
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\[ \mathbf{u} = \text{Displacement vector} \]
\[ \mathbf{f} = \text{Harmonic forcing function} \]

(Note that \( \mathbf{f} \) and \( \mathbf{u} \) are time dependent and varying harmonically so can also be shown as \( \mathbf{f}_0 e^{i\omega t} \) and \( \mathbf{u}_0 e^{i\omega t} \) where \( \mathbf{f}_0 \) and \( \mathbf{u}_0 \) are the amplitude vectors of force and displacement respectively.)

This is the basic equation of steady state motion. It can be solved directly at a single frequency to give the response \( \mathbf{u} \) to the given force \( \mathbf{f} \). This can be extended to give the response over a range of frequencies by repeating the analysis. To accurately predict the response over a wide frequency range may require a large number of frequency steps and so will often use much more computer time than for the other types of analysis described above.

The analysis is usually run in two stages: First the solution of the equation above over a range of frequencies to produce response spectra for any chosen nodes. Second the equation of section 2.2.1 is used to derive the complete stress state at only one or two chosen frequencies, typically the resonance peaks (full calculation of the stress state at every frequency would require excessive computer time).

The results available from this type of analysis are therefore a set of response spectra for nodes of interest (which will characterise the vibration behaviour and show the damped resonant frequencies) and a number of modeshapes and stress states at points of interest in the spectra (generally the resonant frequencies).

This type of analysis has been used particularly for the design and optimisation of the tuned ultrasonic mounting (chapter 5).
2.3 REDUCED DYNAMIC ANALYSIS

One major disadvantage of dynamic analysis (modal and particularly harmonic response) is the computing time and power required for a full analysis. An improved technique has therefore been developed and is available on most finite element programs including Ansys. The purpose of reduced dynamic analysis is to reduce the computing requirements with minimal loss of accuracy. Indeed if computing power is in any way limited (as it invariably is) a full analysis will only run on a coarser model than is possible with reduced analysis and better results will therefore be obtained using reduced analysis. Accuracy of results has been studied and is described in section 2.8.

Reduced modal / reduced harmonic response analysis is based on reducing the order of the matrices and vectors in the equations described previously so that only certain degrees of freedom are included. These are called "master" degrees of freedom or simply "masters". The large mass, stiffness and damping matrices which include every degree of freedom defined in the model are reduced to smaller matrices which include only the master degrees of freedom by processes known as "Guyan reduction" (Guyan [109]) for the stiffness matrix and "static condensation" for the mass and damping matrices. These techniques will not be discussed here but are very well explained in the references.

The choice of which degrees of freedom should be designated as masters is crucial to the accuracy of the analysis. The reduced matrices will most accurately reflect the full model if the masters are at degrees of freedom associated with high mass and/or high compliance. The Ansys program will select the master degrees of freedom automatically unless the user specifies a set of his own. Repeated attempts were made to improve upon the automatic selections without success, so for the analysis work described here the Ansys automatic masters have been used.
2.4 ELEMENT TYPES

The Ansys User Manual [110] lists 77 different element types ranging from simple spars to 3-D hyperelastic solids. It is the responsibility of the user of the Ansys program to decide which element type to use for any application. Often more than one type could be used and each of the possible types will have advantages and disadvantages. For this work six element types have been tried, effectively three pairs. Each pair comprises one type of element with mid-side nodes and one without (section 2.4.4). In practice the results obtainable tend to be similar using element types with or without mid-side nodes. The main differences lie in the selection of:

1) 3-D elements
2) 2-D Plane Stress elements
3) 2-D Axi-symmetric (Axi-harmonic) elements

The advantages and disadvantages of each will be described in turn.

2.4.1 Three-Dimensional Elements (Ansys ref. STIF45, STIF95)

The use of 3-D Isoparametric Solid ("brick") elements is natural for any isotropic solid structure. Other solid elements may also be used e.g., wedges or tetrahedra and these can to some extent be mixed with brick elements where required. Automatic meshing (see section 2.1.1) can usually generate brick elements if the shape of the object is suitable or tetrahedral elements for more general structures. Ultrasonic dies with an approximately square cross section are suitable for meshing with brick elements so these have been used. Occasionally the shape of the internal form may require wedge elements. These are defined as for brick elements but with two pairs of nodes duplicated (note that this is not recommended when using elements without mid-side nodes - see section 2.4.4). Figure 2.07 illustrates these concepts.
The main advantage of a full 3-D model using this type of element is that all possible resonance modes can be found. It is very easy to use a simpler 2-D model (as in the following sections) and fail to predict an important mode of vibration because it is excluded by the assumptions inherent in the model.

The main disadvantage of using 3-D elements is the computing power and analysis time required to complete the analysis. Given a certain fineness of meshing there could be at least four times more degrees of freedom in the 3-D model than in either of the following 2-D models. Hence for the same level of accuracy it is reasonable to assume that at least four times as many master degrees of freedom should be specified (so that they are similarly distributed). Analysis time can be roughly calculated by the cube of the number of (master) degrees of freedom, so a full 3-D model could require very roughly 64 times the analysis time of a 2-D model for similar accuracy. In practice this is seldom relevant because computing power limits the maximum number of degrees of freedom that the program can handle. For Ansys
running on the CarnaudMetalbox MicroVAX a full 3-D model cannot be run with enough masters to give accuracy approaching that obtainable from the 2-D models (see section 2.8).

There is one further option available to reduce analysis time and computing power requirements but it does involve some compromise on the major advantage of the 3-D models i.e. their ability to predict all possible modes. This is to take advantage of symmetry and model only part of the die (figure 2.08). If the uniform radial mode used for these dies was the only one of interest then a thin sector of the die could be modelled. Constraints must be placed on the radial edges simulating the presence of the rest of the die by forcing the radial edges to move only radially. This excludes all harmonic modes except those which have radial antinodes separated by an angle equal to the wedge angle. (Section 3.1 describes the modes of an ultrasonic die and the nomenclature developed.)
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One simplification which does not at first sight appear to exclude any modes is to model exactly half the die (i.e. a wedge as above with a 180° included angle). All the harmonic modes by definition (section 3.1) must have at least two antinodes on opposite sides of the die. However even this simplification does restrict the die. The constraints on the radial faces prevent rotational motion of the die about its own axis and hence exclude the modes designated F0, D0 and FD0 (section 3.1 and figure 3.03).

Modelling only half the die loses some other information also. Imperfections in the die will cause each harmonic mode to appear as a pair of resonances at slightly different frequencies although one of the pair may be difficult to detect. These will correspond to the same mode aligned differently on the die. A full 3-D model does show these mode-pairs but a half model cannot. It may seem strange that a computer model can demonstrate an effect caused by imperfections in materials or manufacturing, but in fact the computer model itself contains corresponding imperfections caused by irregularity of positioning of master degrees of freedom or even rounding errors in the arithmetic. In practice these mode pairs are not usually detectable because the fitting of an ultrasonic transducer promotes one of the pair (aligned with the transducer) and suppresses the other.

Summarising, 3-D elements should be used as a check that important modes of resonance have not been missed by models using other elements, or more generally if computing requirements are of no consequence. Other element types offer more efficient solutions for most work.
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2.4.2 2-D Plane Stress elements (Ansys ref. STIF42, STIF82)

Figure 2.09 shows the use of 2-D elements (plane stress and axi-symmetric) for analysing ultrasonic dies.

These elements are in fact general purpose 2-D solid quadrilateral elements i.e. they can be used for plane stress, plane strain or axisymmetric analysis depending on the options chosen. Axisymmetric elements (obviously quite appropriate for ultrasonic dies) will be covered in the following section. The other option is plane stress, assuming that the die is relatively short in the axial direction so that axial stress can be considered constant through the die length. Variations in die thickness can be modelled by assigning different thickness values to the elements as required.

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**Figure 2.09 - 2-Dimensional Finite Element Models**

- Plane Stress Model (STIF42/82 Elements)
- Axisymmetric Model (STIF29/83 Elements)
In the same way that the 3-D elements can be converted from bricks to wedges by defining duplicate edge nodes these 2-D quadrilateral elements can be converted to triangles by specifying a duplicate corner node. Again this is only recommended for the mid-side node elements (STIF82).

For the radial modes the plane stress assumption can be shown to be fairly good so a model using these elements behaves rather like a 3-D model which has been restrained in the axial direction (this type of element does not include any degrees of freedom perpendicular to the model plane). This then is the main disadvantage with this type of model: the modes of resonance involving significant axial motion will be excluded (this means the torsional and some of the rotational modes).

This type of element requires much less computing power and time than the 3-D element for similar accuracy (on the modes which it is capable of predicting). It has not been used very much because the 2-D axi-symmetric/harmonic element gives better results. The only exception to this is in the study of interactions between different radial modes (see chapter 7). Good die design will eliminate these interactions by ensuring that the resonant frequencies of unwanted modes are well separated from the working frequency (chapters 3 and 4).

2.4.3 2-D Axi-symmetric elements (Ansys ref. STIF25, STIF83)

As mentioned above the 2-D solid quadrilateral elements can also be specified as axi-symmetric elements. There are other more specialised elements however which will perform this function and do much more. These are the axi-symmetric harmonic elements which model axi-symmetric structures but can be set up to allow deflections which vary harmonically around the structure.

These elements are intended for analysing axisymmetric structures subjected
to non-axisymmetric loading (e.g. a shaft in bending) where full 3-D analysis would require excessive computer time. They are also ideal for analysing ultrasonic dies. In this application the important point is that the harmonic variations around the die match the possible resonance modes of an ultrasonic die perfectly.

Figure 2.10 shows the assumed deflections in the structure depending on the setting of an Ansys command option called "MODE" (note that capital letters will be used for the Ansys command to distinguish it from a "mode" of resonance):

MODE 0,0 corresponds to axisymmetric deflections i.e. radial displacement and hoop displacement are constant around the die, tangential displacement is zero.
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MODE 0,1 corresponds to rotational deflections i.e. both radial and axial displacements are zero, tangential displacement is constant around the die.

MODE n,m (where n≥1) corresponds to harmonic deflections with radial and hoop displacements varying as \( \cos (n\theta) \), tangential displacement varying as \( \sin (n\theta) \). Note that although the value of m may be specified it has no significance in this case.

The main advantage of this element type is that by using the MODE setting all possible modes of resonance can be predicted and the analysis time is much less than for a 3-D model to similar accuracy for reasons explained above. This applies even though the analysis must be repeated a number of times with different MODE settings.

The only significant disadvantage of this type of element is that because analyses of different MODE settings are done separately there is no possibility of simulating the coupling between resonance modes with different harmonic numbers. In the design of ultrasonic dies it is desirable to eliminate this coupling but if it is to be analysed then one of the other element types described above must be used.

2.4.4 Elements with Mid-side Nodes

In each of the sections above reference has been made to two Ansys element types (STIF45 / STIF95, STIF42 / STIF82 and STIF25 / STIF83). In each case these correspond to the simple element (corner nodes only) and the more advanced element (with mid-side nodes in addition). Thus for the 3-D elements STIF45 has 8 nodes while STIF95 has 20 and for the 2-D elements STIF42 and STIF25 have 4 nodes while STIF82 and STIF83 have 8 (see figure 2.11).
Comparison of elements with and without midside nodes applies similarly to each of the three element types described above. Inevitably there are advantages and disadvantages in using the elements with mid-side nodes.

Elements with midside nodes can be used to model curved boundaries (e.g. the form on the inside of a die) with a coarser mesh than would be necessary using simple elements. Also a coarser mesh can be used to obtain a similar level of accuracy. However the nodes and elements will usually be generated automatically by the program so the advantages of these differences are minimal.

The simple elements have been shown to be marginally less accurate when extreme accuracy is required but may sometimes be more efficient (i.e. accuracy vs analysis time) in analysing ultrasonic dies as shown in section 2.8. The mid-side node elements have been found to generate non-solvable matrices in reduced analysis with a large number of masters. This causes the
One other important difference becomes apparent when a finite element mesh is produced within a complex shape. This often requires the use of triangular elements (because quadrilateral elements cannot adequately fit the shape). Triangular elements can be produced by specifying one of the nodes twice, but this is not recommended practice when using 4-noded quadrilateral elements because the resulting element is artificially stiff. 8-noded quadrilateral elements can be used as 6-noded triangles without problems. The 3-D elements are similar: 20-noded bricks can be used as 15-noded wedges but 8-noded bricks should not be used as 6-noded wedges.

Generally this work shows that except in some special cases there is little to choose between these two forms of element. Both have been used in producing the results given here.
2.5 APPLICATIONS OF ANALYSIS & ELEMENT TYPES

The aim of this section is to summarise the applications of the different types of analysis and elements.

2.5.1 Static die analysis

Static loading on ultrasonic dies is caused by the interference fit used to hold the insert into the outer die. Static analysis using axi-symmetric or 3-D elements is required. The loading is completely axi-symmetric so there is no advantage in using a 3-D model and the elements could be either STIF25/STIF83 (with MODE set to 0,0) or STIF42 / STIF82 (set for axi-symmetric not plane stress).
The die is modelled by its cross-section as shown in figure 2.12. The nodes on the interface between insert and outer die are duplicated so that no node is common to both. The insert nodes may even be defined in positions just outside the corresponding outer die nodes to accurately represent the interference, although in practice this overlap will be invisible. Care must be taken to ensure that the two mating surfaces are defined by pairs of nodes in exactly corresponding positions (this requires some care when using the automatic meshing function). Each pair of nodes must then be linked using coupling and constraint equations.

Coupling defines that certain degrees of freedom of two or more nodes must have the same displacement value. In this case coupling is used to ensure that the axial (UY) motion of the insert is linked to that of the die outer. At least one of the node pairs must be coupled in this way to prevent the insert from sliding out of the die. Using a fairly high interference it is reasonable to assume (at least as a first assumption) that friction will prevent any relative movement between the mating surfaces so all node pairs could be coupled in this way.

Constraint equations are a more general form of coupling. Again certain degrees of freedom of two or more nodes are linked but now an equation can be defined to specify the nature of the link. To simulate an interference fit an equation is defined for each node pair such that whatever the radial (UX) displacement of the insert node, the displacement of the corresponding outer die node must be greater by an amount equal to the radial interference. i.e.

\[ UX_2 = UX_1 - r \]

where

- \( UX_1 \) = Radial displacement of node 1 (on insert)
- \( UX_2 \) = Radial displacement of node 2 (on outer die)
- \( r \) = Radial interference
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FIGURE 2.13 AXISYMMETRIC MODEL - INTERFERENCE FIT DISPLACED SHAPE

FIGURE 2.14 AXISYMMETRIC MODEL - INTERFERENCE HOOP STRESS
The main results of this analysis are the displaced shape and the stresses (see figures 2.13 and 2.14). Probably the only use for the displaced shape is to predict the amount of shrinkage of the inside diameter of the insert after fitting. This would only be required if the insert was to be pre-machined to very tight tolerances. The stresses are of more general use and these should be studied in conjunction with the stresses calculated by dynamic analysis e.g. modal (see the following section and section 3.8).
2.5.2 Dynamic die analysis

Most of the dynamic analysis work on dies has had two main aims: first to find a design with satisfactory natural frequencies and second to determine the optimum interference fit and maximum working amplitude.

To find the important natural frequencies the die is analysed using modal analysis and 2-D axi-symmetric/harmonic elements. This involves setting up the analysis to be repeated for a series of different MODE values typically 0 1 2 3 and 4. Above MODE 4 the lowest natural frequency is usually well above the range of interest so these natural frequencies are disregarded. Also MODE 0,1 is not usually included because again the lowest natural frequency is higher than the range of interest and is in any case unlikely to interfere with the working of the die. These assumptions have been found to be reasonable for the types of ultrasonic die so far designed. If a new type of ultrasonic die is to be used, or a different working frequency chosen then the assumptions will need to be carefully re-evaluated.
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Figure 2.15 shows an analysis at this stage.

Having found a die design with suitable natural frequencies the next step is to calculate the stresses generated in the die by its vibration in the chosen mode. This could be done using a forced harmonic analysis but Ansys also offers a quicker and easier method involving only a slight modification to the modal analysis above. This involves specifying an amplitude value on which the modes of vibration will be normalised. In the Ansys program this involves specifying a "Seismic excitation" by its direction (radial), type (displacement) and frequency spectrum (usually constant over the frequency range of interest, taking the value of the chosen working amplitude). This option is of course intended for analysis of structures in simulated earthquake conditions but is nevertheless extremely useful in this application.

Figure 2.16 shows one of the results of this analysis for the working mode - the hoop stresses generated in the die by the vibrations.
These results must be studied along with the earlier results of static analysis to determine the optimum interference fit and the maximum safe working amplitude. The structure is undergoing elastic deformation so the stress states predicted by the two analyses (static and modal) can be scaled according to the chosen interference and amplitude respectively. For a successful die design the interference and amplitude must be specified to satisfy a number of conditions as described in Section 3.7.

2.5.3 Dynamic mounting analysis

Analysis of the ultrasonic mounting might appear to be similar to die analysis but in fact the techniques used are very different. The mounting is designed to be a resonant system like the die but unlike the die it has a multitude of natural frequencies in the 20 kHz region. None of the natural modes will correspond exactly to the motion of the die in operation because the mounting is forced to move by the die rather than vibrating freely by itself. (It could be argued that in the process of forcing the mounting the die also ceases to move freely but the die is so much more massive than the mounting that this effect is negligible.) For this reason modal analysis is inappropriate for the mounting. Instead forced harmonic analysis has been used to predict the steady-state response.

The reason for analysing the mounting is to determine how much influence it has on the die vibrations. Optimisation has also been used to minimise this influence. As a measure of influence the force required to move the mounting seems appropriate but the power drain is more useful. Power is calculated from force and displacement as discussed in chapter 5. The displacement is fixed (by the required die movement) so the power really represents the component of force which is in phase with the velocity (or 90° out-of-phase with the displacement).

Chapter 5 describes the analysis and optimisation of the mounting from initial
modal analysis to forced harmonic response (simulating the die motion) calculation of forces and power loss and displaying results for a range of possible mounting geometry.

2.5.4 Summary of Applications

Static analysis of axi-symmetric models has been used to calculate stresses generated in the dies by the interference fit.

Modal analysis has been used to find the natural modes and frequencies of the die. Axi-symmetric harmonic elements have been used for most work but plane-stress and 3-D models have also been used where necessary. Using the "seismic excitation" option modal analysis of axi-symmetric models has also been used to calculate stresses generated in the die by vibration in the working mode.

Harmonic response analysis of an axi-symmetric model has been used to calculate the power losses caused by the mounting.

In addition to this work multiple analyses have been set up to run automatically using parametric analysis as described in the following section. Collecting results for many different geometries of die (chapter 4) and mounting (chapter 5) has led to improved designs and design techniques.
2. Finite Element Analysis

2.6 PARAMETRIC ANALYSIS

The use of parameters in finite element analysis is a very convenient technique for studying alternative designs and finding an optimum. Ansys allows parameter variables to be used in almost all places where a number would be used. Furthermore parameter values can be extracted from the model data or results. Simple arithmetic is possible to manipulate the values and trigonometric functions are also available.

The most common application for parameter variables is in defining model geometry. Any dimension of the model can be defined using a parameter (which must already have been declared). There are two main advantages: First the geometry can be conveniently changed by editing one line of the Ansys input file (changing a dimension not defined by a parameter would require searching through the file to find all occurrences of the dimension). Secondly there is an opportunity to optimise the dimension automatically by running the analysis repeatedly and Ansys has built in optimisation facilities employing standard techniques to optimise a number of dimensions simultaneously (the practical limit is about 10 to 20 variables). This could be used for die design but for most work a simpler technique has been used. This involved running a series of analyses and storing the results from each analysis for subsequent processing and display by other programs.

The following section will describe the use of parameter variables in Ansys. The simple optimisation of die diameters is described in section 3.5 and the later work on developing a general purpose die design technique in chapter 4.
2.6.1 Use of Parameters in Ansys

A parameter variable is defined in one of two ways. For example, to give a parameter X the value 99 the command could be:

*SET,X,99

or more simply:

X=99

Definition of parameters can also use expressions formed from other parameters, constants and arithmetic expressions e.g..

Y=X+20 would give Y the value 99+20=119

Redefinition of parameters is similarly straightforward e.g..

Y=Y-50 would now give Y the value 119-50=69

In the later versions of Ansys the expressions can be specified using FORTRAN conventions. Earlier versions used a limited subset so some examples described here may use apparently unnecessary parenthesis e.g..

Z=((X*5)-(Y/2))+2 X*5-Y/2+2 would not be properly evaluated

When writing these expressions the numeric format can be integer, decimal or scientific. The program treats all numbers as decimal (floating point real). The common trigonometric functions are available if required. Variable names may be up to four characters long.

Extraction of parameter values from the model data or results is done using sorting and the command *GET e.g..

*GET,TOPN,NMAX would give parameter TOPN the value of the maximum node number defined

NSORT,SIGE would sort nodes by Von-Mises stress and
2. Finite Element Analysis

*GET,TOPS,MAX would give parameter TOPS the value of the
maximum stress

For examples of the use of parameters see the example programs described
and listed in appendix 3.
2. Finite Element Analysis

2.7 EXAMPLES

The Ansys program allows for interactive use (each command typed in by the user being implemented immediately) or command-file input (using the same commands stored on file). The Micro-VAX operating system is similar, although with a completely different set of commands. A series of command-file listings with descriptions of the program functions is included in appendix 3.

The programs listed are typical examples demonstrating the use of static, modal and harmonic response analysis, plus the combination of VAX commands to control file handling and multiple analyses. Comment lines are included to assist the understanding of each command. For further details see appendix 3.
2.8 ACCURACY

The analysis of ultrasonic dies can never be perfectly accurate. When comparing FE predictions with measured results, differences may arise as a result of errors within the FE analysis, mismatch between the FE model and the real system and measuring inaccuracy. Some of the possible sources of inaccuracy are:

1) Tolerances on die dimensions
2) Tolerances on material properties
3) Measuring uncertainty (e.g. effect of attaching a transducer)
4) Human error in analysis or measurement
5) Simplifying assumptions used in modelling
6) Inability of model to precisely match reality
7) Calculation methods for fast results

Problems associated with human error are probably inevitable but cannot be studied here. The other sources of inaccuracy may be divided into two broad groups which could be called model-reality mismatch and modelling inaccuracy. Both groups have been studied to determine the reliability of the results obtained from this work, and to ensure that the finite element models are set up for best accuracy within limitations of time and computing power.

The methods used to study accuracy were:

1) Comparison of FE results with real measurements
2) Comparison of FE results with 'exact' solutions
3) Comparison of FE results with other FE results

The results of this work were a set of recommendations for finite element modelling of ultrasonic dies which, if followed, should produce satisfactory die designs.
2. Finite Element Analysis

2.8.1 Comparison with real measurements

Section 6.2 details the results predicted and measured for all ultrasonic dies manufactured. These show that the analysis was generally fairly good but the results must be treated with caution.

There are some particular points to note:

1) The discrepancies are fairly consistent from one die to another. Clearly there are some random influences but there are significant constant errors.

2) If the chosen 20 kHz resonant frequency is to be achieved exactly (or within say 0.05 kHz) then a ‘tuning allowance’ of about 5 mm must be added to the die outside diameter. The die can then be tuned by machining the outside diameter until the measured resonant frequency is acceptable.

3) Unwanted resonant frequencies which are predicted below 20 kHz tend to be fairly accurate but those predicted above 20 kHz are often overestimated by about 0.5-1.0 kHz. This is caused by the combination of analysis errors and the frequency change caused by the transducer. Analysis errors tend to overestimate resonant frequencies while the transducer tends to pull the frequencies towards 20 kHz (its own frequency). Hence below 20 kHz the errors will tend to cancel while above 20 kHz they will accumulate.

2.8.2 Comparison of FE results with "exact" solutions

By making two assumptions it is possible to simplify the governing equations enough to obtain an exact theoretical solution to the die vibration. The two assumptions are axisymmetric motion and plane stress. The solution method is detailed in Appendix 2. It is similar to an analysis by Biddell [38] but also includes a further enhancement which extends the results for a series of concentric hollow cylinders of different dimensions (allowing approximate modelling of die shapes). Even in this type of analysis the results are calculated by series expansions (to obtain the Bessel functions) and so
cannot strictly be said to be exact but any desired degree of accuracy can be achieved.

Hutchinson and El-Azhari [86] developed a series solution for the hollow cylinder which is not limited to plane stress axisymmetric but this is inevitably much more difficult to evaluate, requiring the eigensolution of a matrix equation very much like the finite element solution.

The finite element results were checked against both of these 'exact' solutions for mode-shape and natural frequency. The comparison with the plane stress / axisymmetric solution was inevitably limited by the assumptions made in that analysis while the comparison with the complete series solution was limited by availability of results.

2.8.3 Comparison with series solution

El-Azhari expresses his results in non-dimensionalised form for a wide range of different hollow cylinder geometry. A hollow cylinder similar to the early steel ultrasonic dies was chosen to compare results, but the geometry had to be chosen to precisely match one of the published geometries. The model for comparison was as follows:

Inside diameter = 0.040 m
Outside diameter = 0.160 m
Thickness = 0.050 m
Young's Modulus E = 210 GPa
Density \( \rho = 7800 \text{ kg/m}^3 \)
Poisson's ratio \( \nu = 0.30 \)

The corresponding non-dimensional geometry parameters used by El-Azhari are as follows:
2. Finite Element Analysis

\[ a = \frac{1}{4} \]
\[ h = \frac{5}{16} \]
\[ 4h = 1 \]
\[ (1+a) \]

The non-dimensional frequencies are converted to actual frequencies by multiplying by the shear wave velocity and dividing by the outside radius of the cylinder i.e.

\[ f = \frac{\omega \sqrt{G/p}}{2\pi(0.080)} \]

where \( f \) = frequency / Hz

\( \omega \) = El-Azhari frequency

\( G \) = shear modulus = \( \frac{E}{(1+\nu)} \)

<table>
<thead>
<tr>
<th>MODE TYPE</th>
<th>( \omega ) /kHz</th>
<th>( f ) /kHz</th>
<th>Ansys freq /kHz</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>4 x 4</td>
<td>6 x 6</td>
</tr>
<tr>
<td>R0</td>
<td>2.8543</td>
<td>18.273</td>
<td>18.273</td>
</tr>
<tr>
<td>R1</td>
<td>2.8013</td>
<td>17.934</td>
<td>17.935</td>
</tr>
<tr>
<td>R2</td>
<td>1.7061</td>
<td>10.922</td>
<td>10.923</td>
</tr>
<tr>
<td>R3</td>
<td>3.3812</td>
<td>21.646</td>
<td>21.655</td>
</tr>
<tr>
<td>R4</td>
<td>4.6349</td>
<td>29.672</td>
<td>29.684</td>
</tr>
<tr>
<td>R5</td>
<td>5.6756</td>
<td>36.334</td>
<td>36.349</td>
</tr>
<tr>
<td>T0</td>
<td>1.8050</td>
<td>11.555</td>
<td>11.539</td>
</tr>
<tr>
<td>T1</td>
<td>2.6794</td>
<td>17.153</td>
<td>17.146</td>
</tr>
<tr>
<td>T2</td>
<td>1.1220</td>
<td>7.183</td>
<td>7.174</td>
</tr>
<tr>
<td>T3</td>
<td>2.3358</td>
<td>14.953</td>
<td>14.947</td>
</tr>
<tr>
<td>T4</td>
<td>3.4862</td>
<td>22.318</td>
<td>22.312</td>
</tr>
<tr>
<td>T5</td>
<td>4.5963</td>
<td>29.425</td>
<td>29.420</td>
</tr>
</tbody>
</table>

\( \omega \) = non-dimensional El-Azhari frequency

\( f \) = calculated El-Azhari frequency for FE model

Table 2.1 Comparison of natural frequency predictions
These formulae have been used to prepare the data in table 2.1 which compares the actual frequency extracted from the El-Azhari data with frequency predictions from Ansys for FE models using 8-noded axi-harmonic elements in 4 x 4 and 6 x 6 meshes.

For the radial modes the table shows excellent agreement particularly for the lower-order modes and (as would be expected) for the 6 x 6 element model. Even for the 4 x 4 model with harmonic number 5 the discrepancy is only 15 Hz (0.04 %). For the torsional modes the agreement is again very good but here it appears that the FE results are more accurate than the series solution (both are converging downwards towards the true solution). The series solution could be made more accurate by taking more terms but published data is not available for this in the particular data set chosen.

2.8.4 Comparison with plane stress / axisymmetric solution

For comparison with the plane stress / axisymmetric solution a simple hollow cylinder was chosen to roughly simulate the early solid steel ultrasonic dies (see chapter 3 for more details). The dimensions and material properties were as follows:

- Inside diameter = 40 mm
- Outside diameter = 140 mm
- Thickness = 1 to 70 mm (for FE models which required it)
- Young's Modulus = 209 GPa
- Density = 7800 kg/m³
- Poisson's ratio = 0.28

Note that for a plane stress model the thickness is irrelevant but the FE models which do not inherently include the plane stress assumption must have it defined. A range of thickness was used to ensure that convergence
towards a plane stress solution was achieved (the solution must tend to plane stress as the thickness tends to zero).

Figure 2.17 shows the results of the plane stress / axisymmetric analysis. Predicted lowest natural frequency is 20.108 kHz and the variation in amplitude and radial stress along a radius is shown in the graphs. The finite element results for amplitude and stress distribution are shown as points superimposed on these graphs. These show excellent agreement on displacement results and good agreement on stress. A finite element model using a finer mesh would improve further on these results, but this level of accuracy is certainly adequate for predicting safe working amplitude. By contrast extremely high accuracy is required in predicting the natural frequencies of the die, and this is the subject of the following figures.
Figure 2.18 shows the convergence of the R0 natural frequency estimated by the corresponding FE solution using 8-noded 2-D axi-harmonic elements (for an explanation of element types see section 2.4). Two model meshes were used: 4 radial x 4 axial for die thicknesses from 10 to 70 mm and 4 radial by 1 axial for die thicknesses from 1 to 10 mm. This selection was to prevent the use of elements with excessive aspect ratios (ratio of largest to smallest dimension). Agreement between the two models (at their common thickness 10 mm) is very clear. Convergence as thickness tends to zero is towards a natural frequency of 20.108 kHz, agreeing with the "exact" solution to better than 1 Hz (0.005%).

![Diagram of frequency convergence vs thickness](image-url)
Figure 2.19 shows the convergence of the frequency estimated by the FE solution using 4-noded 2-D axi-harmonic elements. The same two model meshes were used: 4 radial x 4 axial for die thicknesses from 10 to 70 mm and 4 radial by 1 axial for die thicknesses from 1 to 10 mm. While the general trend is the same as for the 8-noded elements the convergence is in this case much less clear and the agreement between the two models (at thickness 10 mm) is poor.

![Diagram showing frequency convergence vs thickness](image)

It was thought that some of this variation might be caused by the changing aspect ratio of the elements as the thickness varied so a further series of tests was run. Firstly the model was fixed at thickness 10 mm and a number of different meshes tested, each with approximately the same number of elements. The meshes were 16 radial x 1 axial, 8 x 2, 5 x 3, 4 x 4, 3 x 5 and 2 x 8 giving a range of aspect ratio from 0.3 to 20. Figure 2.20 shows the variation of predicted natural frequency for these models. The frequency
varies from 19.795 kHz to a maximum of 20.040 kHz when the aspect ratio is approximately unity. Clearly when the best possible accuracy is required the variations caused by aspect ratio are significant (although in many cases an error of about 1% might not be considered important).

FIGURE 2.20 ASPECT RATIO SENSITIVITY - STIF25 ELEMENTS
Secondly a series of models with different thicknesses were tested using meshes chosen from the list above to ensure an aspect ratio of exactly unity. These were 16 x 1 thickness 3.125 mm, 8 x 2 thickness 12.5 mm, 5 x 3 thickness 30 mm and 4 x 4 thickness 50 mm. Figure 2.21 shows the convergence of these results and while it agrees with the "exact" solution better than the earlier results using these elements the agreement is still not as good as was obtained using the 8-noded elements.

Figures 2.22 and 2.23 show similar results from models using finer meshes based on 32 to 36 elements. The meshes used were 36 x 1, 18 x 2, 12 x 3, 8 x 4, 7 x 5, 6 x 6, 5 x 7 and 4 x 8. Figure 2.22 shows a pattern of aspect ratio sensitivity as before. The convergence towards plane stress shown in figure 2.23 shows better agreement than before with the "exact" solution but is still not quite as good as the result from 8-noded elements.
FIGURE 2.22  ASPECT RATIO SENSITIVITY - STIF25 ELEMENTS, FINE MESH

FIGURE 2.23  FREQUENCY CONVERGENCE VS THICKNESS - STIF25 ELEMENTS, UNITY A.R., FINE MESH
Figures 2.24 and 2.25 show the same comparison using 2-D iso-parametric plane stress elements. Here convergence is dependent on mesh size only (for plane stress elements the die thickness is irrelevant). Note that convergence towards the "exact" solution is extremely rapid in the case of the 8-noded (STIF82) elements, but much slower for the 4-noded STIF42's. Also note that inaccuracy introduced by a coarse mesh here causes an overestimate of the natural frequency, in comparison to that shown in figures 2.20 and 2.22, where high or low aspect ratios cause an underestimate of the natural frequency.

![Figure 2.24 Frequency Convergence vs Mesh - STIF82 Elements](image)
Figures 2.26 and 2.27 show the comparison using 3-D iso-parametric elements. The accuracy here is somewhat limited by constraints on computing time and power (hence the use of a $3 \times 3 \times 3$ mesh for the 20-noded STIF95 elements). As might be expected, these graphs show a combination of the features of the earlier 2-D element graphs. The convergence towards a plane stress value with reducing thickness is similar to figure 2.18 but the plane stress value is not the 'exact' 20.108 kHz. Instead the value corresponds to that of the corresponding mesh in plane stress elements e.g.. $4 \times 4$ STIF42 plane stress (4-noded) gives a frequency 20.264 kHz, $4 \times 4 \times 2$ STIF45 (3-D 8-noded) converges to 20.267 kHz.
2. Finite Element Analysis

**Figure 2.26** Frequency Convergence vs Thickness - STIF95 Elements

**Figure 2.27** Frequency Convergence vs Thickness - STIF45 Elements
2. Finite Element Analysis

2.8.5 Comparison with other finite element results

The results described above all refer to full mode-frequency analysis of the die models. Reduced analysis is another option available (as described in section 2.3). This can help to improve the efficiency of the analysis by allowing the use of a finer mesh with reduced analysis time. The results of similar test runs with reduced mode-frequency analysis will now be described. Note that for this work the results for reduced analysis will be compared with the earlier full analysis results rather than the 'exact' solutions. This is firstly to avoid the necessity to test convergence towards plane stress with repeated analysis runs for different die thicknesses, and secondly to allow natural frequencies other than the RO to be checked. The model used for these tests was the same as the previous one except that a constant 50 mm thickness was used, i.e.

Inside diameter  = 40 mm
Outside diameter  = 140 mm
Thickness        = 50 mm (for FE models which required it)
Young's Modulus  = 209 GPa
Density          = 7800 kg/m³
Poisson's ratio  = 0.28

The reference data was taken from the results using full analysis of 8-noded axi-harmonic elements in a 4 x 4 mesh (figure 2.18) because this model has been found to give near perfect agreement with the 'exact' solutions. The natural frequencies of radial modes in the vicinity of 20 kHz were as follows:

Radial axisymmetric (R0) 19.922 kHz
Radial 1st harmonic (R1) 20.551 kHz
Radial 3rd harmonic (R3) 23.930 kHz

Figure 2.28 shows the convergence of predicted natural frequencies towards these values for a reduced model with a 4 x 4 mesh of 8-noded axi-harmonic elements. Note the different scale of this graph compared to the previous
ones - the errors in a reduced model can be very significant. The advantage of a reduced model is in the reduction in analysis time (also indicated on the graph). Note that for certain values of the number of master degrees of freedom the analysis failed so no data is available. This happens when the number of masters is approximately half of the total number of degrees of freedom, or greater. In these circumstances reduced analysis is not really appropriate - the technique is intended for use with much fewer masters than total degrees of freedom - and the analysis failures probably reflect this. For a $4 \times 4$ mesh of 8-noded elements there are 65 nodes, 130 degrees of freedom (axisymmetric) or 195 degrees of freedom (axi-harmonic). Using 80 master degrees of freedom gives fairly good accuracy (better than 1% for the modes of interest) without excessive analysis time or risk of analysis failure.

![Figure 2.28 Frequency Convergence vs Number of Masts - STIF83 Elements](image)

**Figure 2.28** Frequency Convergence vs Number of Masters - STIF83 Elements
Figure 2.29 is the equivalent graph for a 7 x 7 mesh of 4-noded axi-harmonic elements (the mesh size chosen to give an almost identical number of nodes (64 in this case) and degrees of freedom. The graphs show that for these elements the failure of analysis is not a problem but for large numbers of master degrees of freedom the natural frequencies may be significantly underestimated. Best results are obtained again with the number of masters about one third to one half of the total number of degrees of freedom. At this point, accuracy and analysis time are similar to the previous results for 8-noded elements.

![Graph showing frequency convergence vs number of masters for different element meshes.](image)

**Figure 2.29** Frequency convergence vs number of masters - STIF25 elements

Given that the number of master degrees of freedom may be limited by practical considerations such as analysis time it is useful to study the effect of different element meshes on the results for a fixed number of masters. Figure 2.30 shows results for a model using 8-noded axi-harmonic elements which is limited to 40 masters. Again failed analyses have reduced the amount of data available for display but it is clear that a finer mesh does not always produce
better results. In this case a 3 x 3 mesh is the best option (with a total of 40 nodes this will have 80 or 120 total degrees of freedom - again 1/3 to 1/2 that number of masters).

FIGURE 2.30 FREQUENCY CONVERGENCE VS NUMBER OF ELEMENTS - STIF83 ELEMENTS
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Figure 2.31 is the equivalent to 2.30 for 4-noded elements. Again there is an optimum mesh size (4 x 4 giving 25 nodes) but note how rapidly the errors increase for a non-ideal mesh size.
2.8.6 Accuracy recommendations

From the results described here some guidelines for die analysis can be suggested:

Best accuracy is achieved using full analysis and elements with midside nodes.

Using elements without midside nodes a finer mesh is required and the element aspect ratio should be approximately unity.

Axisymmetric-harmonic elements offer accurate results for all modes of vibration in minimum analysis time, but preclude the possibility of interactions between modes of different harmonic number.

To reduce analysis time reduced modal analysis is recommended with the number of master degrees of freedom approximately equal to the number of nodes. The best accuracy to be expected using this method is approximately 1 % on natural frequencies.
This chapter describes the application of finite element analysis to the design of ultrasonic dies, with consideration of modes of vibration, performance (ie. ability to maintain high-amplitude vibrations in the chosen mode) and practical aspects of die design and manufacture.
3.1 MODE NOMENCLATURE

For any vibrating system there are numerous possible modes of vibration. In order to describe the characteristics of the system it is essential to have a means of identifying each mode uniquely. In the case of ultrasonic dies there was no available system of identifying modes but the "axi-harmonic" elements (see section 2.4) used in the commercially-available finite element packages did offer a useful approach. The system of nomenclature which was developed to describe the resonant modes of ultrasonic dies will now be described.

The mode is described by means of two features. Firstly the movement or distortion of the die cross section (i.e. whether it moves radially, rotates, stretches axially etc.) and secondly the variation in that movement / distortion around the die (in a pure mode constant or varying harmonically). Figure 3.01 shows this diagrammatically. The following sections describe how these features are classified.

![Diagram of mode nomenclature](image)

**FIGURE 3.01 - MODE NOMENCLATURE**
3.1.1 Mode Type

The description of the movement of the die cross section is inevitably somewhat subjective but for most of the modes in the low frequency range (i.e. close to or below the working frequency) the following descriptions can be related to the actual modes without any doubts.

**Radial** - The cross section of the die moves radially in and out. This movement is usually accompanied by some axial distortion - as the cross-section moves out it becomes thinner axially and as the section moves in it becomes thicker. This is caused by the Poisson's ratio of the material. There is usually a little distortion radially also. The inside of the die often moves less than the outside so as the section moves out it may become thicker radially and vice versa.
Axial - The cross section of the die stretches axially. This is usually accompanied by some radial distortion - as the die stretches axially it becomes thinner radially and vice versa (this is the Poisson's ratio effect again). Note that this mode can be distinguished from the radial mode described above firstly by the relative amplitudes of the radial and axial motion (the names used naturally refer to the larger component) and secondly by the radial motion on the inside and outside surfaces of the die (at this one cross section). In the axial mode the motion of the inside and outside surfaces is usually quite different and they may even move in opposite directions.

Torsional - The cross section of the die rotates about a point somewhere near its centre. Distortion of the cross section tends to be minimal but depends on the shape of the cross section and the materials used. The name torsional was chosen because this mode is analogous to the torsional vibrations of a square-section shaft bent into a circle.

Figure 3.02 shows each of these mode types.
Rotational - In contrast to the "torsional" mode described above this mode involves essentially a rotation of parts of the die about the central axis. There are three main types:

Face rotational - The two faces of the die rotate in opposite directions.

Diameter rotational - The inside and outside surfaces of the die rotate in opposite directions.

Face-Diameter rotational - A combination of the two above ie. each end of the inside diameter rotates in the same direction as the opposite end of the outside diameter.

Figure 3.03 shows these rotational mode types.
Axial Bending - Only found on dies which are relatively thin in the axial direction, this mode involves the cross section bending so that the inside and outside move axially in one direction while the centre moves in the opposite direction.

Radial Bending - This is equivalent to the axial bending mode but found in dies which are relatively thin in the radial direction and involves radial movement of the two faces of the die with a corresponding opposite radial movement in the centre of the cross section.

Figure 3.04 shows diagramatically the axial and radial bending mode types.
This is obviously not a complete list of all possible modes of vibration (there are an infinity) but includes all of the modes which would be found at the lower end of the frequency spectrum for a typical ultrasonic die.

For convenience the mode type may be abbreviated to one or two letters i.e.:

- R  Radial
- T  Torsional
- F  Face rotational
- D  Diameter rotational
- FD Face-diameter rotational
- RB Radial Bending
- AB Axial Bending

### 3.1.2 Harmonic Number

The harmonic number describes the variation of amplitude (and hence strain and stress) around the die at any selected point on the cross section. This is based on the theory (strictly true only for rings of small cross section - see Den Hartog [82] or Timoshenko [83]) that for a axisymmetric structure in free vibration the amplitude must be constant around the die or vary harmonically as follows:

<table>
<thead>
<tr>
<th>Mode Type</th>
<th>Amplitude Formula</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Radial</td>
<td>$k_1 \cdot \cos (n.\theta)$</td>
<td>where $k_1 , k_2 , k_3$ are constants</td>
</tr>
<tr>
<td>Axial</td>
<td>$k_2 \cdot \cos (n.\theta)$</td>
<td>depending on mode</td>
</tr>
<tr>
<td>Tangential</td>
<td>$k_3 \cdot \sin (n.\theta)$</td>
<td>$\theta$ is angular co-ordinate, $n$ is harmonic number</td>
</tr>
</tbody>
</table>

This offers a number of possibilities according to the value of the harmonic number $n$:

1) $n \geq 1$ The radial and axial amplitudes vary harmonically around the die. Tangential amplitude also varies harmonically but is out of phase with the other components i.e. tangential amplitude is a maximum when radial and axial amplitudes are zero and vice versa.
2) $n = 0$ The radial and axial amplitudes are constant around the die. Tangential amplitude is zero throughout.

Note that there is a third option not predicted by the equations shown above but from another set of equations which are equally valid in which the sine and cosine terms are reversed. Option 1 above will be the same using either set.

3) $n = 0$ The radial and axial amplitudes are zero throughout the die. Tangential amplitude is constant around the die.

In the system of nomenclature used here the harmonic number follows the mode type to fully specify the mode eg:

**R0** specifies a radial axisymmetric mode

**T3** specifies a torsional third harmonic mode

Figures 3.05 and 3.06 show the effects of harmonic number on radial and torsional modes respectively.
3.1.3 Mode Families

There is one further complication in the specification of vibration modes. In many cases the mode description above covers not just one mode but a whole family eg. the axial bending mode described is the lowest-frequency member of a family of diaphragm modes. This also applies to the radial-bending modes and more importantly to the radial mode as described in Section 4.5. Figure 3.07 shows the principle applied to radial-bending and axial-bending modes. A number is again used to specify the member of the family, or left blank to indicate the simplest, lowest-frequency member of the family. To avoid confusion with the harmonic number this "mode complexity" number (if used) is placed before the mode type identifier:

0RB0 Simplest axi-symmetric radial bending mode (as RB0)

1RB0 Axi-symmetric radial bending mode with one extra bend and a central nodal ring
2AB2 Axial bending (diaphragm) mode with 4 radial nodal lines (because harmonic number is 2) and 2 circumferential nodal lines (because there are 2 extra bends).

Clearly the range of possible modes of vibration for any ultrasonic die (or other hollow cylinder) is extremely complex. Fortunately for most dies with a relatively square cross section these families of modes are not generally relevant because the frequency range of interest covers only the simplest modes and a limited range of harmonic numbers.

3.1.4 Selection of Working Mode

Historically several different modes of vibration have been used to assist metal-forming processes. Prof. D.H. Sansome, a prominent researcher in this field during the 1960's and early 1970's tried radial and axial excitation of the die and also axial excitation of the plug (which could be used in combination)
in tube drawing processes [52]. Research was also carried out in Eastern Europe along similar lines, notably by Tisza [39] who described a radial mode die which was driven axially with the shape of the die converting some of the axial motion to radial.

The general consensus of this work was that radial motion of the die was most beneficial, offering the possibility of friction reduction by separating the surfaces and forging the material down (i.e. changing its shape by means of numerous hammer blows). A discussion of reasons for friction reduction and a review of the published prior art appear in sections 1.2 and 1.3.

A further consideration in the selection of a working mode of vibration is the siting of the transducer. For correct functioning of the ultrasonic equipment it is imperative that the surface to which the transducer is attached moves uniformly along the transducer axis. Any other motion will cause the transducer to bend and modern piezo-electric transducers will not tolerate this for long.

Figure 3.08 shows these requirements.

One mode of those described above offers a reasonably uniform radial motion of the inside surface and acceptable transducer positioning on the outside surface - this is the R0 (radial axisymmetric) mode. Note however that with this transducer positioning the other radial modes (R1, R2, R3 etc.) will be equally excited.
3.1.5 Summary

Each mode is described by its mode type (abbreviated to one or two letters) and its harmonic number, eg.

R0 - Radial axisymmetric (chosen mode for forming dies)
T1 - Torsional first harmonic
A0 - Axial axisymmetric (chosen mode for plastic welding)
F2 - Face rotational second harmonic
FD0 - Face-diameter rotational axisymmetric
Where the mode type belongs to the same family as those described above but corresponds to a higher degree of distortion (within the cross section) an initial number may also be used to describe this eg.

1AB0 - Axial bending (diaphragm) axisymmetric involving one more bend in the cross section than the basic mode.

1R0 - Radial axisymmetric mode with a nodal ring so that while the inside diameter of the die contracts the outside diameter is expanding.

Figures 3.01 to 3.07 show diagrams of all the modes described here.

For best performance in metal-forming applications the R0 mode has been found to be most useful (although the 1R0 mode might also be considered for certain applications - see section 7.2). All other modes are unwanted and may interfere with the correct functioning of the die.
3.2 FREQUENCY BEHAVIOUR

In most simple vibrating systems the natural frequencies of successive modes can be expected to increase more or less regularly with the mode number. In particularly simple systems (such as a taut string) there will be a direct relationship between mode number and natural frequency. In ultrasonic dies there are significant variations from this type of frequency behaviour which can adversely affect the performance of the die. Therefore the frequency behaviour of a typical ultrasonic die will be discussed in detail.

In this section the higher degrees of distortion of the cross section (shown for example in the '1R0' mode) will not be considered. Instead the families of the same mode-type and increasing harmonic number will be described, particularly the family of radial modes (R0, R1, R2, R3 etc.).
3. Ultrasonic die design

Figure 3.09 shows a typical distribution of natural frequencies for the radial and torsional modes of an ultrasonic die. Note that from harmonic number 2 upwards the natural frequencies do increase fairly regularly as would be expected. The natural frequencies for harmonic number 0 and 1 are out of place however, much higher than would be expected from the trends shown by the rest of the modes. These two natural frequencies often appear close to that of the R3 mode. There is a similar pattern of natural frequencies in the family of torsional modes also.

This means that the unwanted R1 and R3 modes will appear at natural frequencies close to that of the chosen R0 mode. Depending on how close they are, and other factors, this may lead to problems which prevent the die working properly. There are two particular problems which will be discussed in the following sections: Mode coupling and mode switching.

3.2.1 Mode coupling

This is a well known phenomenon in many vibrating systems having two or more degrees of freedom. In forced vibration (with damping) there is an interaction between two modes of resonance which are near neighbours in the frequency range. The responses of the system both appear as combinations of the pure modes shapes. This modal combination is accompanied by a shift in the resonant frequencies such that the two frequencies separate slightly.

This type of behaviour has been noted in several early ultrasonic dies which were designed without the benefit of finite element analysis (see section 6.2).

If mode coupling occurs between the R0 mode and another (usually a radial harmonic mode) then the result is two modes which both have some characteristics of the two pure modes. Thus the required "pure" R0 mode ceases to exist. Some coupling is acceptable however, provided that the R0...
mode is still identifiable and the distortion is small (say less than 5%).

Figure 3.10 shows how response curves for an ultrasonic die can change as some property is varied (eg as the outside diameter is reduced during tuning). Two modes appear initially at resonant frequencies $f_1$ and $f_2$ (well separated). As the diameter is reduced the frequencies change in a fairly consistent manner and may move closer together. Using this consistency it could be predicted that for a particular outside diameter the frequencies would appear close together at $f_3$ and $f_4$ but the actual frequencies are found further separated at $f_5$ and $f_6$. The modeshapes for these resonances will be found to be combinations of the pure modes.

![Figure 3.10 - Mode Coupling - Frequency Behavior](image)
Figure 3.11 shows a possible combination mode showing elements of both R0 and R3 modes.

This effect is associated with damping and also depends on the separation of the (theoretical undamped) natural frequencies concerned. The forming process inevitably introduces some damping which is dependent on the process. Ultimately the damping will be limited by the power capacity of the ultrasonic generator (i.e. the vibration amplitude used will be chosen to draw maximum power from the generator). Therefore an initial frequency separation can be specified which will ensure that mode coupling is limited to an acceptable level. About 0.5 to 1.0 kHz is usually adequate for this purpose.
3. Ultrasonic die design

3.2.2 Mode switching

This is related to the automatic frequency control of the ultrasonic electrical generator (frequency converter) which continuously adjusts the driving frequency to maintain resonance (the resonant frequency will change depending on temperature, die loading etc.). Various techniques have been used by the different equipment manufacturers to achieve this ranging from positive feedback oscillators to phase-locked-loop systems - see Sonkin [78]. The simpler systems may be prone to lose the resonance in the chosen mode and instead switch to maintaining resonance in a nearby unwanted mode.

This has been observed with one ultrasonic system (switching from the required R0 mode to the unwanted R3) on certain dies while the same dies work perfectly on another system.

The conditions under which this problem will appear are obviously much more complicated than those which control mode coupling but from practical work an acceptable frequency separation can be determined. This has been found to be about 1.5 to 2.0 kHz, ie. 7 to 10% of the operating frequency. Note that preventing mode switching (if the simpler generator is to be used) requires a greater frequency separation than is required to prevent mode coupling.
3.3 FREQUENCY ANALYSIS

The first priority then in the design of a new ultrasonic die is to promote the required mode of vibration and exclude all others. This can be achieved by ensuring that the required (R0) natural frequency matches the working frequency of the ultrasonic equipment to be used (here 20 kHz) and that no other natural frequencies appear close by. The purpose of frequency analysis is to predict the natural frequencies of all modes in the frequency range from 0 to about 25 kHz or more.

At this initial analysis stage only the natural frequencies and mode-shapes are evaluated. Mode-frequency (KAN,2) analysis is used, usually with axisymmetric-harmonic elements to model the die (but see also section 3.5 which describes shaped dies unsuitable for this type of element). The reasons for choosing different types of analysis and element were described in sections 2.2 to 2.4. Some typical results are shown in figures 3.12-3.15.

The important points to note here are:

1) The working (radial axisymmetric) mode appears at a natural frequency of 20.158 kHz. This is slightly too high and will need to be reduced by increasing the outside diameter of the die.

2) One unwanted radial mode natural frequency is very close to the working frequency - the R3 at 20.095 kHz. It is not unusual in the design of 20 kHz ultrasonic dies to find that the R3 frequency appears close to the R0 but this is particularly close and therefore likely to cause problems of mode coupling and / or switching. This design will need to be changed to increase the separation.

3) One "torsional" mode (involving essentially a rotation of the cross-section) also appears at a natural frequency close to the working frequency - the T4 at 20.378 kHz. This is unlikely to interfere with the operation of the die because the movement of the die's outer surface in this mode is not compatible with the transducer motion.
3. Ultrasonic die design

FIGURE 3.12 FREQUENCY ANALYSIS - RADIAL AXISYMMETRIC (RD) MODE

FIGURE 3.13 FREQUENCY ANALYSIS - RADIAL 1ST HARMONIC (R1) MODE
3. Ultrasonic die design

**Figure 3.14** Frequency analysis - radial 3rd harmonic (R3) mode

**Figure 3.15** Frequency analysis - torsional 4th harmonic (T4) mode
The results described above can be summarised in a tabular form as in table 3.01.

<table>
<thead>
<tr>
<th>RADIAL MODES</th>
<th>TORSIONAL MODES</th>
</tr>
</thead>
<tbody>
<tr>
<td>MODE</td>
<td>FREQ /kHz</td>
</tr>
<tr>
<td>R0</td>
<td>20.158</td>
</tr>
<tr>
<td>R1</td>
<td>15.599</td>
</tr>
<tr>
<td>R2</td>
<td>9.800</td>
</tr>
<tr>
<td>R3</td>
<td>20.095</td>
</tr>
<tr>
<td>R4</td>
<td>28.208</td>
</tr>
</tbody>
</table>

TABLE 3.01 PREDICTED NATURAL FREQUENCIES - ALUMINIUM DIE OUTER

If there was no problem with the unwanted natural frequencies then the next step would be to adjust the outside diameter of the model to place the R0 frequency at 20 kHz (to any desired degree of accuracy). However the unwanted R3 frequency appears so close to the same frequency that it will inevitably remain a problem unless a more significant design modification is made.

The simplest modification to make at this stage is to change one of the materials. High strength aluminium alloy is used for low-cost experimental tools but for its superior strength and resistance to damage titanium alloy is preferred in production tooling. If the same geometry is used with a titanium alloy die outer then the mode shapes are again similar but the frequencies are as shown in table 3.02.
Note that although the R3 and T4 frequencies still appear close to 20 kHz the R0 frequency is now significantly lower. This offers the possibility that in "tuning" the R0 mode up to 20 kHz the unwanted modes will be moved to more convenient frequencies. The next step therefore is to adjust the die geometry (specifically the outside diameter) to bring the R0 frequency closer to 20 kHz. A simple rule of thumb which is fairly accurate for many ultrasonic dies is that a 1 mm reduction in outside diameter causes an increase of approx. 0.1 kHz in the R0 natural frequency. (This rule is also used in a slightly modified form for fine tuning the frequency of a die after manufacture - see section 3.8). In this case the R0 frequency is to be increased by 1.2 kHz so the outside diameter must be reduced by 12 mm. The modified model gives apparently identical modeshapes but different frequencies as listed in table 3.03.
### TABLE 3.03 PREDICTED NATURAL FREQUENCIES - TITANIUM DIE OUTER (AFTER TUNING)

<table>
<thead>
<tr>
<th>MODE</th>
<th>FREQ /kHz</th>
<th>MODE</th>
<th>FREQ /kHz</th>
</tr>
</thead>
<tbody>
<tr>
<td>R0</td>
<td>20.028</td>
<td>T0</td>
<td>11.246</td>
</tr>
<tr>
<td>R1</td>
<td>17.245</td>
<td>T1</td>
<td>17.944</td>
</tr>
<tr>
<td>R2</td>
<td>9.822</td>
<td>T2</td>
<td>6.758</td>
</tr>
<tr>
<td>R3</td>
<td>20.978</td>
<td>T3</td>
<td>14.811</td>
</tr>
<tr>
<td>R4</td>
<td>30.000</td>
<td>T4</td>
<td>22.750</td>
</tr>
</tbody>
</table>

The R0 (working) mode is now at a fully acceptable frequency (although further fine-tuning could bring it even closer to 20 kHz). The ideal 2 kHz separation from unwanted radial modes has not been achieved. The separation with the R3 frequency is approx. 1 kHz. This would probably be enough to prevent mode coupling but there would be a danger of mode switching depending on which ultrasonic generation system is used. Some further separation of the unwanted frequencies might be achieved by a minor change to the die cross section. Alternatively the "shaped" die designs described in section 3.5 would offer another possible solution.
3.4 OPTIMISATION OF DIE DIAMETER

The process of adjusting the die diameter to place the R0 frequency at 20 kHz is straightforward and therefore suitable for computerisation. The technique used Ansys parameters to control the optimisation process. The optimisation process was done outside Ansys using VAX command files. It would also have been possible to implement it entirely within Ansys using the built-in optimisation package but the command-file approach was preferred because this would allow monitoring of the optimisation process while it was running in batch mode.

The aim was to analyse a proposed new die, find its natural frequency, hence calculate the required change in die diameter and analyse the die again at this new diameter. This process would be repeated until the natural frequency was 20 kHz (to any desired degree of accuracy).

Using VAX command files this process was implemented using four files:

1) Ansys Run-file. This is a VAX command file containing the instruction to run Ansys followed by all the Ansys commands required to define the die (with its outside diameter defined by a parameter), run the analysis, process the results to extract the required resonant frequency and write that frequency out to a data file.

2) Results data file written as above, read by...

3) Main command file. This is a VAX command file including commands to execute the Ansys run file, read the results from the data file produced and calculate the frequency error. If the frequency is outside the specified tolerance band then the program would calculate the required correction to the outside diameter and edit the Ansys run file to include the new value before repeating the above procedure. At some stage in this loop one further file is created or updated...

4) Trace file. This is used to tell the user what the program is doing when it is
running in batch mode. This file is initially created when the main command file is first run and then updated each time a new outside diameter is calculated.

Example 2 in appendix 3 shows a typical Ansys run file. Example 5 is the main command file used for all dies.
3.5 SHAPED DIES

In many cases the frequency of one of the unwanted modes of vibration will appear close to the working frequency. It may be possible to correct this by changing one or more of the non-critical dimensions (e.g., die inner thickness, die outer thickness, interface diameter etc.). Alternatively if other materials can be used then a different combination will give a different set of natural frequencies. In practice it may be very time-consuming to check all possibilities and the work described in Chapter 4 was aimed at drastically reducing this checking time. In some cases the options are so limited by various circumstances that no die design with satisfactory natural frequencies can be found (see section 4.5).

For these cases another approach was required. All analysis to this point had assumed a hollow cylindrical die. (In reality the die has a small flat machined on its outside surface to accommodate the ultrasonic transducer but this is small enough to have little effect on the natural modes and frequencies). It was thought that by slightly modifying the cylindrical shape the unwanted harmonic natural frequencies could be modified without greatly affecting the working R0 mode of vibration.

This section details work done to investigate this possibility.

3.5.1 Evaluating die shapes

The outside surface is the only practical place to introduce a non-cylindrical shape. The inside surface is dictated by the product (it may or may not be round, but cannot be changed to suit the ultrasonic system). The interface surface could in theory be non-cylindrical but in practice the accuracy required to match two polygonal surfaces would prove prohibitively expensive. Therefore the design study was limited to dies with non-cylindrical outside surfaces and these were modelled using Ansys.
In order to maintain a near-cylindrical shape and ensure easy manufacture the die shapes studied initially were combinations of arcs (based on the original cylinder) and flats. The number, positions and sizes of these flats was varied and the effect on the unwanted natural frequencies was estimated. The finite element models for this work were based on the plane-stress (STIF42/82) elements. Figure 3.16 shows some of the results in tabular form.

Clearly the different shapes have different effects on the die vibrations. There are two important changes:

1) The unwanted natural frequencies are shifted relative to the working R0 frequency. The R0 frequency may itself be moved by the change to a 'shaped' die but this can be corrected by a change in the overall size of the die. In figure 3.16 changes in the relative positions of the unwanted R1 and R3 frequencies greater than 1 kHz have been highlighted in bold type. The shapes giving rise to these frequency shifts are potentially the most useful.

2) The shape of the R0 mode is distorted so that the surface motion is no longer uniform and radial. Note that if this happens then the definition of the R0 mode (section 3.1) is no longer valid. Equally the other modes will also be distorted and again the definition will not accurately describe them. Nevertheless the mode nomenclature described in section 3.1 is still usable because these distorted modes are directly traceable back to the original 'pure' modes.
<table>
<thead>
<tr>
<th></th>
<th>R0</th>
<th></th>
<th>R1 ALIGNED VERTICAL</th>
<th></th>
<th>R2 ALIGNED VERTICAL</th>
<th></th>
<th>R3 ALIGNED VERTICAL</th>
<th>CHANGE IN SEPARATION</th>
</tr>
</thead>
<tbody>
<tr>
<td>ROUND</td>
<td>20.44</td>
<td>20.94</td>
<td>20.96</td>
<td>11.61</td>
<td>11.63</td>
<td>24.49</td>
<td>24.46</td>
<td></td>
</tr>
<tr>
<td>ROUND, ONE 30° FLAT</td>
<td>20.47</td>
<td>20.89</td>
<td>21.12</td>
<td>11.62</td>
<td>11.63</td>
<td>24.56</td>
<td>24.43</td>
<td>-0.08 +0.11 +0.04 -0.06</td>
</tr>
<tr>
<td>3 X 60° FLATS EQUI-SPACED</td>
<td>20.68</td>
<td>20.41</td>
<td>20.42</td>
<td>11.76</td>
<td>11.79</td>
<td>25.64</td>
<td>24.76</td>
<td>-0.77 -0.80 +0.91 +0.06</td>
</tr>
<tr>
<td>3 X 60° FLATS+ONE 30° FLAT</td>
<td>20.78</td>
<td>20.41</td>
<td>20.42</td>
<td>11.78</td>
<td>11.79</td>
<td>25.65</td>
<td>24.81</td>
<td>-0.87 -0.90 +0.82 +0.01</td>
</tr>
<tr>
<td>2 X 80° FLATS EQUI-SPACED</td>
<td>20.65</td>
<td>17.75</td>
<td>25.65</td>
<td>12.15</td>
<td>11.63</td>
<td>26.00</td>
<td>25.20</td>
<td>-3.40 +4.46 +1.30 +0.73</td>
</tr>
<tr>
<td>2 X 70° FLATS EQUI-SPACED</td>
<td>20.69</td>
<td>18.61</td>
<td>24.14</td>
<td>11.96</td>
<td>11.65</td>
<td>25.58</td>
<td>24.93</td>
<td>-2.58 +2.91 +0.84 +0.22</td>
</tr>
<tr>
<td>2 X 60° FLATS EQUI-SPACED</td>
<td>20.62</td>
<td>19.39</td>
<td>23.00</td>
<td>11.82</td>
<td>11.64</td>
<td>25.16</td>
<td>24.66</td>
<td>-1.73 +1.84 +0.49 +0.02</td>
</tr>
<tr>
<td>2 X 60° FLATS 60° INCLUDED</td>
<td>20.55</td>
<td>21.70</td>
<td>19.92</td>
<td>11.69</td>
<td>11.77</td>
<td>25.25</td>
<td>24.60</td>
<td>+0.65 -1.17 +0.65 +0.03</td>
</tr>
<tr>
<td>4 X 60° FLATS EQUI-SPACED</td>
<td>21.05</td>
<td>21.00</td>
<td>20.99</td>
<td>11.97</td>
<td>11.65</td>
<td>25.50</td>
<td>25.40</td>
<td>+0.45 +0.48 +0.40 +0.32</td>
</tr>
<tr>
<td>6 X 60° FLATS EQUI-SPACED</td>
<td>21.56</td>
<td>22.41</td>
<td>22.34</td>
<td>11.93</td>
<td>11.91</td>
<td>26.06</td>
<td>25.22</td>
<td>+0.35 +0.24 +0.45 -0.36</td>
</tr>
<tr>
<td>4 X 60° FLATS 60° INCLUDED</td>
<td>21.07</td>
<td>23.43</td>
<td>19.80</td>
<td>11.79</td>
<td>11.86</td>
<td>25.81</td>
<td>24.96</td>
<td>+1.86 -1.81 +0.69 -0.13</td>
</tr>
<tr>
<td>3 X 70° FLATS EQUI-SPACED</td>
<td>20.82</td>
<td>19.85</td>
<td>19.89</td>
<td>11.86</td>
<td>11.88</td>
<td>26.56</td>
<td>25.06</td>
<td>-1.47 -1.47 +1.69 +0.22</td>
</tr>
<tr>
<td>2 X 60° FLATS 30° INCLUDED</td>
<td>20.58</td>
<td>22.54</td>
<td>19.48</td>
<td>11.69</td>
<td>11.76</td>
<td>24.91</td>
<td>24.89</td>
<td>+1.46 +1.64 +0.28 +0.29</td>
</tr>
<tr>
<td>2 X 70° FLATS 30° INCLUDED</td>
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<td>23.14</td>
<td>18.74</td>
<td>11.72</td>
<td>11.87</td>
<td>25.28</td>
<td>25.26</td>
<td>+2.02 -2.42 +0.61 +0.62</td>
</tr>
<tr>
<td>ROUND, ONE 60° FLAT</td>
<td>20.64</td>
<td>19.99</td>
<td>21.86</td>
<td>11.72</td>
<td>11.63</td>
<td>24.78</td>
<td>24.55</td>
<td>-1.15 +0.68 +0.09 -0.11</td>
</tr>
</tbody>
</table>

FIGURE 3.16 - SHAPED DIES - EFFECTS OF VARIOUS SHAPES
The best die shape is one which causes minimum distortion of the R0 mode while increasing as far as possible the separation between the R0 frequency and the nearest unwanted frequency. This means that the best shape will depend on which unwanted frequency is closest to the working frequency and whether it appears above or below (i.e., whether it should be moved up or down). Furthermore, the other unwanted frequencies will be affected by the change to a shaped die and it is possible that one of these will become a problem as a result of the change. This means that the best shape will depend on the relative positions of the working natural frequency and all the unwanted ones. Almost independently of the shape the degree of shaping (e.g., the size of the flats) can also be varied to provide adequate frequency separation with the minimum distortion of the R0 mode.

In figure 3.16 two shapes are identified as particularly useful. Figure 3.17 shows these shapes (based on flats of 60° included angle). Other shapes also affect the unwanted frequencies of course but these two show the greatest effect for any given flat size. The variation of frequency shift with flat size (defined by the included angle) is shown in figure 3.18. These shapes will now be considered in turn.

3.5.2 Two-flat design

Note that the shapes labelled A and B in figure 3.17 are essentially the same except that the transducer is attached to a flat of shape A where it is fitted to an arcuate portion of shape B (although a small flat is required on the arc to accommodate the transducer as is used on the round dies). The important point is that this shape causes the R1 frequency to split into two— one at a higher frequency and the other at a lower frequency. The transducer filters out one of these modes while supporting the other. If the transducer is fitted on one of the flats then the lower frequency is supported while with the transducer on one of the arcs the higher frequency is supported. Note that both shapes cause a small upward shift of the R3 frequency but little change in the R0.
3. Ultrasonic die design

**Figure 3.17 - Shaped Dies - Best Options**

- **A**
  - 2-flats (transducer on flat)
  - Lowers R1
  - Raises R3

- **B**
  - 2-flats (transducer on arc)
  - Raises R1
  - Raises R3

- **C**
  - 3-flats (transducer on arc)
  - Raises R3
  - Lowers R1

**Figure 3.18 - Variation of Radial Mode Frequencies with Flat Size**

- **A**
  - 2-flats (transducer on flat)
  - Lowers R1
  - Raises R3

- **B**
  - 2-flats (transducer on arc)
  - Raises R1
  - Raises R3

- **C**
  - 3-flats (transducer on arc)
  - Raises R3
  - Lowers R1
Clearly if the unwanted R1 mode of vibration appears at a frequency close to the working frequency then either of these shapes could be used to move it up or down. In practice the shape would be chosen depending on whether the initial natural frequency was above or below the working frequency.

The reasons for the usefulness of the 2-flat design are not straightforward. To understand why the flats affect the natural frequencies as they do it is essential first to understand the stresses in the die while it is vibrating in the R1 mode. Figure 3.19 shows the R1 modeshape. Because the amplitude varies as cos(θ) around the die it is clear that while at one radial antinode the cross-section moves inward at the other it will be moving outwards.
In global terms there are two parts of the die - the two radial antinodes - moving in the same direction at the same time. To maintain the centre of gravity in a fixed position (a necessary condition for free vibration) there must be a corresponding amount of momentum in the opposite direction. This opposing movement happens at the nodes of radial motion, which are also antinodes of tangential motion (see section 3.1.2). While the central hole remains almost undistorted the outer surface of the die is heavily distorted by this tangential motion. This causes a stress state in the die which varies from tension/compression and bending at the radial antinodes to shear at the radial nodes.

From this the reasons for the changes in natural frequencies become clearer. When the flats are positioned at radial antinodes (shape B in figure 3.17) where the local stresses are tensile/compressive and bending they make the die more flexible and hence reduce its natural frequency. Positioned at the radial nodes the flats increase the shear strain at these points because the same amount of shear must take place over a smaller distance. This effectively stiffens the die and increases its natural frequency in the R1 mode.

3.5.3 Three-flat design

Based on this principle a 6-flat design should have a similar effect on the other main unwanted mode - the R3. Figure 3.16 shows that this is true but the change in the frequency separation between the working R0 mode and the unwanted R3 mode is too small to be useful. This is because the flats (and the arcuate areas between them) do not concentrate the stiffness changes enough to have the desired effect on the R3 mode. The spacing between nodes and antinodes is naturally three times less for the R3 mode than for the R1. To concentrate the mass and stiffness changes more a smaller flat could be used but this also results in a smaller change in the die stiffness which has less effect. Using slots rather than flats is a possibility
although there is a danger of excessive stress concentrations. This is discussed in section 3.5.4.

The 3-flat design (figure 3.17 'C') does have a useful effect on the R3 frequency but the reasons for this are more complicated than for the 2-flat design. Figure 3.20 shows that the R3 mode has been distorted by the shape change so that the amplitude at the flats is less than at the arcuate parts of the die, although there are antinodes in both places.

If the inward movement of the die at the flats does not match the outward movement at the arcs then the die's perimeter must stretch to provide this extra movement. Similarly when the arcs are moving inwards more than the flats move outwards there must be an overall compression of the die. The extra tensile and compressive stresses generated by this distortion will effectively stiffen the die, causing an increase in the R3 frequency.

Note that only one transducer position is shown for this shape, unlike the
3. Ultrasonic die design

?-flat shape where two transducer positions gave markedly different results. In this case the alternative transducer position (on a flat) gives similar frequency modifications but less frequency shift for any given flat size. Therefore it offers no advantages over the position shown.

3.5.4 Slotted design

As an alternative to the 6-flat design small grooves might be made in the outside surface rather than flats (figure 3.21) to provide maximum removal of material in minimum space. The graphs of natural frequency vs slot depth show that this design can significantly reduce the R3 frequency relative to the R0. At the same time the R1 frequency is also reduced relative to the R0. Note that the effect on the R3 frequency is similar to the effect of the 2-flat design on the R1 - the mode is split into two at very different frequencies. This time however only the lower frequency is useful because the upper one follows the R0 closely and offers no advantage.

![Graph of natural frequency vs slot depth](image)

**Figure 3.21 - 6-Slot Die - Effects on Frequencies**

- Test die is ID 40, OD 140 steel cylinder
- Slots are 10 mm wide, equi-spaced
In the lower R3 mode the antinodes are aligned on the slots, so to select this mode (and filter out the other) it will be necessary to attach the transducer at a slot. This naturally presents practical difficulties with the thread for the transducer stud and furthermore the tangential motion of the die adjacent to the slots would cause severe friction problems.

An alternative design which overcomes these problems uses 5 slots at 60 degree intervals, with a conventional transducer flat in place of the sixth slot. This modifies the frequencies in a similar way and does offer a practical design option. However as shown in figure 3.22 the stress concentrations at the slots, particularly the two closest to the transducer, are significant and may limit the working amplitude for a die of this type (see stress analysis - section 3.7)
3.6 DIE MATERIALS

An ultrasonic die for metal forming requires a number of diverse material properties. The metal-forming operation requires extreme hardness and wear resistance while the stresses generated by the action of the ultrasonics require a tough, strong and fatigue-resistant material. In addition the die must not absorb too much energy from the vibrations so the material must have low acoustic losses (measured by logarithmic decrement or quality factor Q). Unfortunately there is no material which combines all these properties. In particular the requirements for extreme hardness and toughness / fatigue resistance tend to be mutually exclusive. The solution is of course to combine two materials, with a hard, wear-resistant inner die fitted into a tough, fatigue resistant outer. This however introduces one further requirement of the materials used: They must be compatible to allow the transmission of ultrasonics across the interface. Figure 3.23 summarises these requirements.
It is common practice in the plastic welding industry to use high-strength alloys of aluminium and titanium for ultrasonic components. Here the requirements are mainly for toughness, fatigue strength and low acoustic loss although wear resistance may also be important especially when welding filled polymers (eg. glass filled). Adequate wear resistance can usually be achieved by surface treatment such as hard anodising on aluminium. This would not be adequate to withstand a metal forming process but these materials are ideal for the die outer which does not require extreme wear resistance.

No other materials have been found which match the properties of these two for the die outer. Other grades would also be suitable but the particular alloys which have been used are:

- Aluminium L168 - Aircraft standard high strength alloy.
- Titanium Ti64 - General purpose high strength alloy.

Published material properties for all materials listed are given in Appendix 4.

Note that although the fatigue strength of the titanium alloy is much greater than that of the aluminium alloy (about double) this does not relate directly to the real capabilities of the material in this application. Because titanium has a higher density than aluminium it is also subject to higher dynamic stresses at any given vibration amplitude and frequency. Generally a titanium component is capable of withstanding slightly higher amplitudes than an aluminium one but will cost more and absorb more energy.

For the inner die the choice of materials is more conventional. For deforming steel there are many suitable materials but metal-bonded cermets are often preferred for maximum life. Of these probably the most common is tungsten-carbide in a cobalt matrix. Unfortunately this presents problems for ultrasonic dies. The very high modulus and density combine to make this material very difficult to deform elastically (especially considering the relatively low modulus and density of the proposed outer die materials -
aluminium and titanium). This can be expressed by the "acoustic impedance" defined as the square root of the product of modulus and density. A large difference between the acoustic impedances of the materials for the inner and outer dies will lead to a large variation in amplitude across the die cross section. That is not to say that a tungsten carbide inner could not be used in an ultrasonic die but for good performance it will be necessary to use titanium for the outer and design the inner to have a relatively small cross-section. Depending on the shape of the product to be produced this may not be possible for necking dies. In the case of the necking dies for aerosols and beverage cans the size of the inner die was found to be too great to allow the use of tungsten carbide material.

Instead another bonded cermet was tested and proved successful (although full production trials are still required to determine life). This is titanium carbide in a martensitic steel matrix, supplied by Thyssen Fine Steel under the trade name Ferro-titanit Nikro 292. Like tungsten carbide - cobalt this material aims to combine the hardness and wear resistance of a cermet with the toughness and tensile strength of a metal matrix. This particular grade is intended for use in ultrasonic tools and has been used in the plastic-welding industry when a particularly abrasive material is to be welded (the relatively large power loss caused by the acoustic damping of Ferro-titanit makes it impractical for most applications).

Engineering ceramics are also becoming more readily available and one has been tested with good results. This is Syalon - a name which derives from its chemical composition (silicon, aluminium, oxygen, nitrogen). It is effectively a silicon nitride ceramic with aluminium oxide added to improve toughness by distorting the material structure.

A third option for lightly loaded or experimental (short-life) tools is a tool steel. EN 41 has been used because of its ability to take a hard nitride surface treatment. Using plasma nitriding the tools can be finish machined while in the soft state with only polishing required after treatment. For a little extra
wear resistance a coating of titanium nitride can be added. While not offering
the same wear resistance as the options above this is a simple, low-cost
material which has been used with some success.

Nikro 292 - Steel-Titanium Carbide cermet for inner die
Syalon 101 - (Relatively) tough ceramic for inner die
Steel EN41 - Nitriding Steel for lightly loaded or experimental tools

Published material properties for all materials listed are given in appendix 4.
3.7 STRESS ANALYSIS

Having arrived at a die design which gives satisfactory frequency performance the next step is to ensure that it will have an adequate life. These dies are highly stressed by the high-amplitude vibrations so it is essential to match the die design and the operating amplitude to prevent premature failure.

The reasons for using two-piece dies were described in the previous section. The design criteria which determine the life of this type of ultrasonic die are:

- Material of inner die
- Material of outer die
- Die dimensions
- Interference fit
- Operating amplitude

Of these, the first three have already been chosen to suit the application and to obtain satisfactory results from the mode-frequency analysis. The last two may be specified to ensure that the die gives the best possible performance combined with acceptable life. Performance is related to the operating amplitude (a greater amplitude can generally be expected to give a greater reduction in friction) while the life will be determined to a large extent by the stress state within the die materials.

A number of conditions for a satisfactory die life are listed below:

A) The maximum compressive stress in the die insert must not cause failure.

B) The maximum tensile stress in the die outer must not cause failure.

C) The combination of static and dynamic stresses must not cause fatigue failure.

D) The radial stress at the interface must never be tensile (to prevent "chattering").
E) The shear stress at the interface must never overcome friction (any internal sliding will cause energy losses and rapid wear).

Note that when adding stresses to obtain the maximum tensile and compressive values the dynamic stresses must be taken in the worst case i.e. the values shown by the dynamic analysis may be reversed.

In practice conditions D and E are almost equivalent, and the design is dictated by these and condition C. The choice of a suitable interference and working amplitude would involve an iterative procedure as follows:

1) Choose an initial amplitude value.
2) Evaluate dynamic radial stress at interface.
3) Static compressive radial stress must exceed this (condition D above).
4) Choose an interference value which will give required radial stress.
5) Consider fatigue of the die outer under the combination of static and dynamic stresses implied by the chosen values.
6) If fatigue failure is likely then choose a lower amplitude value and start again.
7) If fatigue failure is very unlikely and a higher amplitude is desirable then try a higher value and start again.
8) If the risk of fatigue failure is acceptable and the amplitude is adequate then check all the above conditions before accepting the design.

Inevitably the difficulty here lies in predicting the likelihood of fatigue failure. This is particularly difficult because the materials chosen for the die outer (aluminium and titanium alloys) tend to have no clearly defined fatigue limit (stress below which fatigue will not take place). The best technique is to consider the results of earlier designs (see chapter 6) and err on the side of caution.

Figures 3.24 to 3.27 show the type of stress analysis result which might be
used for this work. Figure 3.24 shows the radial stress generated by R0 vibration at an amplitude of 11.4 microns. At the interface, this must be smaller in magnitude than the radial stress due to the interference fit shown in figure 3.25. If this is satisfactory then the hoop stresses due to the interference fit (figure 3.26) and vibrations (figure 3.27) must be checked to evaluate the risk of fatigue.
3. Ultrasonic die design

FIGURE 3.24 DYNAMIC RADIAL STRESS IN A VIBRATING DIE (RO MODE)

FIGURE 3.25 STATIC RADIAL STRESS DUE TO INTERFERENCE FIT
3. Ultrasonic die design

FIGURE 3.26 STATIC HOOP STRESS DUE TO INTERFERENCE FIT

FIGURE 3.27 DYNAMIC HOOP STRESS IN A VIBRATING DIE (00 MODE)
3. Ultrasonic die design

3.8 DIE MANUFACTURE AND TUNING

The procedure used for the manufacture of ultrasonic dies will now be described. Note that because of uncertainties about material properties and analysis accuracy a tuning operation is required after manufacture.

3.8.1 Manufacturing

Typically two parts will need to be manufactured - the insert and the die outer. The insert is designed to suit the necking process and with an outside diameter to maintain a suitable interference fit (see section 3.7). The length will have been specified to suit the process and / or any special requirements of the ultrasonics. The die outer will have been designed with a suitable bore diameter to maintain the interference and length to suit the ultrasonics (usually slightly greater than the transducer diameter). The outside diameter for manufacturing purposes will be the theoretical diameter plus a tuning allowance (typically 5 mm but depending on the level of confidence in the FE results). The die outer may be made integral with a tubular mounting (see chapter 5) or with a counterbore in the back face into which one can be fitted. (If neither option is adopted then some other method of mounting the ultrasonic system will be required.)

Conventional manufacturing techniques are used for the manufacture of these components. The die outer (aluminium or titanium) can be turned (either conventional or CNC) to the finished dimensions. The manufacturing process for the insert depends on the material chosen. Tool steels can be turned to near the finished shape and finish-ground after heat treatment. In many applications EN41 and ferro-titanit (see section 3.6) can be finish-turned before careful heat treatment and require only repolishing afterwards. Ceramics (and Syalon which although not strictly a ceramic behaves very much like one) are usually turned to near-finished shape in the "green" state (before firing) with allowances on the dimensions for shrinkage.
After firing all surfaces must be finish ground and polished. For grinding Syalon mild steel form wheels (CNC turned) plated with diamond grit have been used successfully.

3.8.2 Assembly

Stress analysis as described in section 3.7 typically indicates that the interference fit required between the two parts of the die is a "heavy press fit". As the name suggests this interference requires a very large force to assemble the parts. This is undesirable because of the risk of damage to the interface surfaces (particularly in the soft die outer) which would give rise to an uneven contact and power losses at the interface. Therefore it is recommended that the insert is shrink fitted i.e. the die outer should be heated and/or the insert cooled before assembly.

There are limitations on this. The alloys of aluminium and titanium are age-hardening i.e. they are held in an unstable equilibrium at room temperature. In time some components form precipitates in the material matrix and increase the material hardness. The ageing process is almost static at room temperature but as the temperature is increased the rate of ageing increases rapidly. Although increased hardness would be a desirable feature the associated embrittlement is certainly not. These alloys are supplied aged to the optimum hardness so further ageing is not desirable. To prevent this the temperature of the die outer should be limited to about 200°C maximum.

In aluminium alloy with its high coefficient of thermal expansion (typically 22 ppm/°C) this will often be enough to provide a clearance for assembly. For example on a 54 mm bore diameter the interference will be about 0.10 mm and the expansion caused by a 175°C temperature rise (assuming 25°C room temperature) is 0.21 mm. Titanium and its alloys have much lower coefficients of expansion (typically about 9 ppm/°C). Therefore heating a
A titanium die may not provide enough clearance to easily fit the insert. (In the example above if the die outer was made from titanium alloy then the thermal expansion would be only 0.07 mm.) In this case the insert may also be cooled. Cooling to approx. minus 96°C can be achieved fairly easily and cheaply using liquid nitrogen (taking care to avoid thermal shock particularly if the insert is made from Syalon). In the example above this cooling would provide an extra 0.13 mm clearance for a steel or ferro-titanit insert (11 ppm/°C) or 0.04 mm for a Syalon insert (3 ppm/°C).

If enough clearance is achieved then it should be possible to simply drop the insert into the die outer but it is wise to set up the outer in a press first so that force can be applied if necessary (figure 3.28). If the insert does "stick" then it must be pushed into place very quickly before the temperatures equalise, reducing the clearance even further.
3.8.3 Tuning

After allowing the die to cool (or warm in the case of the insert) the natural frequency can be tested. Note that frequency testing is not possible before assembly because the natural frequency is determined by the whole assembly. Methods of measuring the natural frequencies and identifying modes are described in chapter 6.

Usually the RU natural frequency is below the required range because of the tuning allowance added to the outside diameter at the design stage. If at this stage the RU natural frequency is significantly above 20 kHz then something has gone seriously wrong and the die outer must probably be scrapped.

The amount of material to be removed from the die outer may be estimated using a simple formula. In general for a 20 kHz die a 1 mm reduction in the outside diameter will cause the RU natural frequency to rise by about 0.08 kHz. Note the difference between this formula and that used for the tuning of finite element model ultrasonic dies (section 3.3 / 3.4) where the same 1 mm reduction diameter was expected to cause a frequency rise of 0.10 kHz. The reason for this difference is that measurements of the actual die are made with a transducer attached. The transducer, itself tuned to 20 kHz, tends to pull the overall resonant frequency towards 20 kHz, and this reduces the apparent effect of changing the die diameter.

Having found the initial RU frequency the above formula can be used to estimate the required reduction in the outside diameter. However the outside diameter should not be machined to the new diameter immediately. Instead the die should be machined by about a half to two thirds of the expected amount and then rechecked. This is to allow for unusual behaviour and measurement / machining errors which might cause the die to be machined too far, and scrapped. The process is repeated until the RU frequency is within the specified range (typically 20 ± 0.1 kHz).
SIMPLIFIED DIE DESIGN PROCEDURE

The technique for ultrasonic die design described in the previous chapter was based on trial-and-error optimisation of the die. There are two possible problems with this approach: the user might miss a good design and / or waste time on unnecessary analysis runs.

An alternative procedure for the design of ultrasonic dies was developed to overcome these problems for many common dies. This is based on a large number of finite element analyses of simplified ultrasonic die designs. The results have been collated and recorded as a series of contour maps. An engineer wishing to design a new ultrasonic die can consult these contour maps and get an idea of the likely problems and potential solutions before starting analysis. In many cases a new ultrasonic die could be designed by reference to the contour maps without any FE analysis.
4. Simplified die design procedure

4.1 PRINCIPLES

This section describes the design requirements of an ultrasonic die and shows how these can be satisfied using the new design procedure.

4.1.1 Design Requirements

(For a fuller explanation of the requirements of a high-power ultrasonic system see Chapter 1 - particularly section 1.2)

For any die to vibrate with significant amplitude it must be part of a resonant system. The system comprises mechanical parts (transducer, die and mounting - see figure 4.01) and electrical parts (a power amplifier) all of which must be individually tuned to the same resonant frequency.

![Diagram of ultrasonic system mechanical parts](image-url)
4. Simplified die design procedure

The electrical components and piezo-electric transducer are proprietary items supplied to work at a frequency of 20 kHz (higher frequency components are available but 20 kHz provides maximum amplitude and convenient tuned dimensions). The mounting is also a tuned section designed to work at 20 kHz and described in detail in Chapter 5. This leaves the ultrasonic die which must be designed to suit the process and also must resonate at 20 kHz.

There are several possible modes of vibration (described in detail in Chapter 3) and the designer must ensure that the vibration mode at 20 kHz will give the maximum benefit. For the processes studied in this research an axisymmetric radial (R0) mode has proved the best. This mode involves the whole die expanding and contracting radially in a uniform way.

Other resonances may appear close to 20 kHz with unsuitable modes of vibration (e.g., the R3 mode which involves the die deforming into a 3 lobed shape). If this happens the automatic frequency control of the ultrasonic generator (used to track small variations in resonant frequency) may switch to the wrong resonance, leaving the die vibrating in the unwanted mode. To avoid this "mode switching" it is important to ensure that other resonances are well separated from the 20 kHz working frequency.

The required separation varies according to the unwanted mode of vibration. Some modes couple well to the motion of the driving transducer (those which involve mainly radial movement at the transducer attachment point). These modes (radial modes, particularly R1 and R3) are particularly likely to cause mode switching. From experience on this project a frequency separation of 1.5 - 2.0 kHz is necessary to be sure to avoid mode switching. Other modes, which do not couple well to the transducer, can be tolerated at smaller frequency separations, down to approximately 0.5 - 1.0 kHz or less.

Summarising, for a high-power ultrasonic die there are several requirements which must be satisfied if the vibration characteristics are to be correct:
4. Simplified die design procedure

1) The R0 (radial axisymmetric) mode must appear at a resonant frequency of 20 kHz.

2) The well-coupled harmonic modes (especially R1 and R3) must appear at a resonant frequency outside the range 18 - 22 kHz.

3) The poorly-coupled modes should preferably appear at a resonant frequency outside the range 19 - 21 kHz.

4) The geometry parameters (diameter of hard die insert, outside diameter of die, thickness, etc) should fall within convenient limits. ie Manufacturing the die must be a practical proposition.

4.1.2 Principles of Simplified Design Procedure

The design procedure is based on approximating any new die to a generalised shape with some variable dimensions. The generalised die could be as shown in figure 4.02.

![Figure 4.02 Generalised Model Ultrasonic Die](image)
4. Simplified die design procedure

This is a die in two parts - a thick cylinder (representing the hard inner die) fitted inside a slightly longer thick cylinder (representing the outer die).

This model has six variable dimensions:

- **IR** - Inside Radius.
- **SR** - Shrink-fit Radius.
- **OR** - Outside Radius.
- **TP** - Thickness of Inner Die (Pellet).
- **TB** - Thickness of Outer Die (Bolster).
- **PP** - Position of Inner Die (Pellet).

The time required to analyse all possible combinations over the range of these variables would be prohibitive, so the number of variables must be reduced. The radial resonant frequencies (usually the important frequencies) are not very sensitive to variations in die thickness, so the variables TP and TB have been taken as constants:

- Thickness of Inner die TP = 38 mm
- Thickness of Outer die TB = 45 mm

Normal design practice requires that the inner die is positioned centrally within the outer. Therefore the variable PP is fixed at \((TB - TP) / 2\) ie:

- Position of Inner Die PP = 3.5 mm

Furthermore, the outside radius is not a design variable - it must be chosen to provide a resonant frequency 20 kHz in the RO mode. An iterative procedure within the analysis automatically generates the value for OR to achieve this.

Having removed these four variables (TP, TB, PP and OR) there are now only two design variables required to describe the die geometry: **IR** (inside radius - usually fixed by the product required) and **SR** (shrink-fit radius) as shown in figure 4.03. This is convenient for displaying results because any calculated result can be displayed against the two independent design variables using a contour map. The program calculates a number of results, each of which must be displayed on a separate contour map.
4. Simplified die design procedure

**FOUR INDEPENDENT VARIABLES:**
- 2 GEOMETRY VARIABLES: IR, SR
- +2 MATERIAL VARIABLES: MATP, MATB

**ONE DEPENDENT VARIABLE:**
- GEOMETRY VARIABLE OR

**THREE CONSTANTS:**
- GEOMETRY VARIABLES TP=38, TB=45, PP=3.5

**FIGURE 4.03 SIMPLIFIED GENERALISED MODEL ULTRASONIC DIE**

Results calculated are:

Outside radius OR (to tune the die for RO at 20 kHz).

Resonant frequencies R0, R1, R2, R3, R4 (radial modes)

T0, T1, T2, T3, T4 (torsional modes)

(The contour plot of resonant frequency R0 is not generally useful because this value has already been set to 20 kHz by adjusting the outside radius OR. However, it does provide a visual check on the correct functioning of the program. Because the program includes a tolerance of +/- 200 Hz on a resonant frequency the actual value can vary above and below 20 kHz, so 20 kHz contours can appear apparently at random over the R0 contour plot. If any other contours appeared on the plot then this would indicate that the iterative routine to optimise R0 had failed).

As described above, 11 contour maps are produced to predict the performance of a generalised ultrasonic die over a range of possible geometry. However, the material properties of the two components (inner and
outer) also have a very significant effect on performance - both outside radius and resonant frequencies. Therefore, for every additional combination of possible material properties, a further 11 contour maps are required. If any combination of materials was possible then this would require a huge number of contour maps. In practice there are few materials suitable for the design of ultrasonic dies, and certain grades have been found to give best results. The selection of materials is described in more detail in Chapter 3. For this work five materials have been considered:

Aluminium L168 (Aircraft alloy. Usually used for outer die)
Titanium Ti64 (General purpose alloy. Usually used for outer die)
Syalon 101 (Toughened ceramic. Used for inner die only)
Nikro 292 (Steel-Titanium Carbide. Usually used for inner die)
Steel EN41 (Nitriding Steel. Usually used for inner die)

Each material has advantages and disadvantages, and any combination might be chosen for a new application.

To allow for any likely future material choice, the properties of these five materials have been incorporated in the analysis. Eleven sets of analyses have been completed (for every reasonable combination of the five materials) producing 11 contour maps for each analysis (121 contour maps in total). For any future dies not using one of these material combinations, the analysis can be repeated with new material properties. The process of data generation (described in section 4.4) is well automated and requires little user input but much computer time. Using the CMB MicroVax II ("Freds") the full program is best run over a weekend.

4.1.3 Summary of Theory

Any ultrasonic die can be characterised by the two materials from which it is made and two geometry variables. These define a single point on each of eleven contour maps. By reading the contour value at this point the user can
predict the performance of the proposed design. Data has been collected for eleven combinations of five materials. If the design requires some other combination of materials, then more data can be generated using the computer programs described in section 4.4. The contour maps displaying design information for each of the eleven material combinations are shown in appendix 5. For the purpose of demonstration, figures 4.05 - 4.07 and 4.09 - 4.11 are duplicates for two material combinations.
4.2 INSTRUCTIONS FOR DESIGNING A NEW ULTRASONIC DIE

This section gives instructions for using the contour maps to design an ultrasonic die for a new application. It is assumed that the inside surface of the die is defined by the process and that the pellet and bolster are to be made from one of the following combinations of materials:

<table>
<thead>
<tr>
<th>INNER DIE</th>
<th>OUTER DIE</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) Syalon 101</td>
<td>Aluminium L168</td>
</tr>
<tr>
<td>2) Syalon 101</td>
<td>Titanium Ti64</td>
</tr>
<tr>
<td>3) Nikro 292</td>
<td>Aluminium L168</td>
</tr>
<tr>
<td>4) Nikro 292</td>
<td>Titanium Ti64</td>
</tr>
<tr>
<td>5) Steel EN41</td>
<td>Steel EN41</td>
</tr>
<tr>
<td>6) Steel EN41</td>
<td>Aluminium L168</td>
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<tr>
<td>7) Steel EN41</td>
<td>Titanium Ti64</td>
</tr>
<tr>
<td>8) Aluminium L168</td>
<td>Aluminium L168</td>
</tr>
<tr>
<td>9) Aluminium L168</td>
<td>Titanium Ti64</td>
</tr>
<tr>
<td>10) Titanium Ti64</td>
<td>Aluminium L168</td>
</tr>
<tr>
<td>11) Titanium Ti64</td>
<td>Titanium Ti64</td>
</tr>
</tbody>
</table>

4.2.1 Determine the Design Variables

The real die must be related to the simplified generalised model as shown in figure 4.03. An average value for the inside radius must be determined from the real shape. The actual value of shrink-fit radius may be used but it is useful also to define a range of permissible values for SR (to permit some fine-tuning of the design).
4. Simplified die design procedure

4.2.2 Choose the Materials

Decide which material combination(s) would be suitable for the application. The procedure described below (4.2.3 - 4.2.5) should be carried out for each possible combination.

4.2.3 Check the Outside Radius

Given values for IR and SR mark a point on the contour map of outside radius corresponding to the new design. By interpolating between the contour lines read off the predicted value of OR. Check that this value is acceptable in the application. Some combinations of materials and geometry produce a die which is unusually large, and may not fit in the space available. Other combinations may require an outside diameter so small that the bolster is too thin to accommodate the flat and stud to which the transducer is attached (see figure 4.01). In some cases the value of OR indicated on the contour map is the same as the value of SR chosen. This indicates that it is not possible to produce a die to this design with the R0 mode at 20 kHz. This applies particularly to designs with a large inside radius - say greater than 35 mm (diameter >70 mm). It may be possible to design a die with a different mode at 20 kHz to give the same forming-force reduction, but this is not within the scope of this design procedure (see Section 4.5.4).

If a range of possible values is available for the shrink-fit radius SR, then this can be represented by a short vertical line on the contour map. A range of possible values of OR is read as before.

4.2.4 Check Radial Modes

As described in Section 4.1.1 the radial harmonic modes are unwanted resonances which are well coupled to the transducer movement. If one is present in the range 18-22 kHz then the vibration performance of the die may
be severely impaired. Using the contour maps the frequencies for modes R1, R2, R3 and R4 can be easily checked. Of these, only R1 and R3 are usually found at a frequency close to 20 kHz. Harmonics higher than R4 are of little interest because the frequencies are well above 20 kHz.

Mark a point on the R1 contour map corresponding to the IR, SR values and read off a value for the R1 resonant frequency in kHz. Repeat for R2, R3 and R4. It is important that every frequency is well separated from 20 kHz. If any frequency is within +/- 1 kHz (ie, in range 19-21 kHz) then the die should be re-designed (either by changing the value of SR or the materials to be used). Frequencies separated by 1-2 kHz (18-19 kHz or 21-22 kHz) represent a risk of poor performance, and in this case the die should preferably be re-designed, although if this is not possible then the die may still function correctly. If all frequencies are separated by at least 2 kHz (ie, outside the range 18-22 kHz) then the radial modes should cause no problem. Past experience suggests that particular care should be taken to separate the R3 mode, and to avoid resonances just above 20 kHz.

If a range of possible values is available for SR, then again these can be represented by a vertical line. By moving along this line it may be possible to increase the frequency separation for a problem mode (but beware - this may also reduce frequency separation on another mode).

4.2.5 Check the Torsional Modes

Torsional modes involve a rotation of the die cross-section and so are not easily excited by the transducer. These modes are, therefore, poorly coupled to the driving force and seldom cause problems.

Nevertheless, some distortion of the pure R0 mode has been observed which appeared to be attributable to a T1 mode, and so it is believed preferable to separate the torsional-mode frequencies from 20 kHz as for radial modes.
Proceed as before to find the predicted resonant frequencies in the T0, T1, T2, T3 and T4 modes using the relevant contour maps. Frequency separation is less important here - less than +/- 1 kHz indicates some risk, but if all frequencies are outside this range (19 - 21 kHz) then there should be no problem.

4.2.6 Revise and Recheck

For many proposed ultrasonic die designs, the above procedure will reveal one or more problems which make modifications necessary. By observing the slope of any contour map which indicates a problem it is possible to predict how the geometry should be modified to improve the design. However, all other contour maps must be rechecked for the revised design to ensure that the modification has not caused new problems.

If a satisfactory design cannot be found within the geometry constraints imposed by the application, then there are two other options:

1) Change materials and restart the design analysis at Section 4.2.3
2) Use a die with the outside surface machined to a non-cylindrical shape, as described in section 4.5.3 and section 3.5.

4.2.7 Accurate FE Analysis

It is important to remember that the results obtained from the contour maps are based on a series of approximations, particularly:

1) The simplified generalised model is a crude approximation to the real die.
2) The Finite Element Analysis can never be perfectly accurate.
3) The automatic die tuning sets the outside radius of the die to 20 kHz plus or minus one per cent.
4) The results are interpolated from the contour maps allowing for inaccuracies in reading.

Despite these approximations the results produced by this technique can be very good as shown by the examples described in section 4.3 below.

If the fit of the simplified generalised model to the proposed die is not good, or possible problems are indicated by the contour maps then it is advisable to analyse the proposed die using conventional FEA and modelling the die geometry fully before manufacture. It is the responsibility of the designer to decide whether this is necessary, or whether other actions should be taken to improve safety margins (eg. A 'tuning allowance' of 5 mm is usually added to the predicted outside diameter to obtain the dimension for initial manufacture - see section 3.7. This might be increased, particularly if the outside diameter then became a stock material size.) If in doubt FEA of the specific die will usually give more accurate results than the contour maps. Accuracy of finite element models is discussed in more detail in section 2.8.
4. Simplified die design procedure

4.3 EXAMPLES

The following two examples illustrate the design analysis procedure described in Section 4.2.

4.3.1 Design of a Die for 45 mm Aerosol Necking

Figure 4.04 shows the design of a typical die for necking welded aerosol cylinders from 45 mm diameter to 31 mm diameter (the main purpose of this project).

Also shown in figure 4.04 is the generalised model used to approximate the real die, showing the choice of design variables IR and SR.
Simplified die design procedure

Suppose the preferred materials for this application are Ferro-titanit Nikro 292 (for the inner die) and titanium alloy (for the outer). The design (now summarised as the values of IR and SR) must be checked using the contour maps for Nikro-Titanium, i.e. figures 4.05 - 4.07.

Referring to figure 4.05 the contour map of Outside Radius, OR, for Nikro-Titanium has been marked with a cross representing the design IR = 19.0, SR = 27.0. Interpolating between the contour lines of OR = 80 and OR = 75 gives a value of approximately OR = 77 mm. This value appears reasonable and convenient (outside diameter of die will be approximately 154 mm).

Also shown on figure 4.05 is the contour map of resonant frequency R0 for Nikro-Titanium. This shows a pattern of 20 kHz contour lines (as described in section 2.2) and indicates that the computer optimisation of OR has succeeded in maintaining the R0 frequency at 20 kHz.

Figure 4.06 shows the contour maps of resonant frequency for modes R1 to R4 for Nikro-Titanium. The proposed design is marked as before on the map for R1 and the value of the R1 frequency (interpolating between the contour lines) is approximately 18.5 kHz. Therefore, the frequency separation is 1.5 kHz (i.e., less than the recommended 2.0 kHz), and this implies some risk of poor performance.

The contour map of R2 frequencies shows that for all possible designs (within the calculated ranges for IR and SR) the R2 resonant frequency is very low - in the range 9 - 14 kHz. Therefore, the R2 mode should cause no problems.

The contour map of R3 frequencies is marked with the proposed design as before. Interpolating again, this gives a frequency of approximately 23.0 kHz for the proposed design. This is acceptable. Repeating this procedure for the R4 frequency and the torsional mode frequencies mapped in figures 4.05 (T0) and 4.07 (T1-T4) completes the analysis of this die design.
4. Simplified die design procedure

Figure 4.05 OR, R0 & T0 contour plots for Nikro - Titanium
Figure 4.06  R1, R2, R3 & R4 contour plots for Nikro - Titanium
4. Simplified die design procedure

Figure 4.07  T1, T2, T3 & T4 contour plots for Nikro - Titanium
The results are summarised below.

Titanium - Nikro 292    IR = 19, SR = 27
OR = 77 mm            Acceptable
R0 = 20.0 kHz         Acceptable
R1 = 18.5 kHz         Not ideal but probably acceptable
R2 = 12.2 kHz         Acceptable
R3 = 23.0 kHz         Acceptable
R4 = 31.0 kHz         Acceptable
T0 = 11.2 kHz         Acceptable
T1 = 18.7 kHz         Acceptable
T2 = 7.1 kHz          Acceptable
T3 = 15.0 kHz         Acceptable
T4 = 22.5 kHz         Acceptable

All results except the R1 frequency are fully acceptable. Returning to figure 4.06, it can be seen that the trend of the R1 contour map is towards lower frequencies for larger values of SR. To move the R1 frequency to a fully acceptable 18.0 kHz, SR must be increased to 30.0. However, this modification will also change all other values so it is essential to re-check.

Titanium - Nikro 292    IR = 19, SR = 30
OR = 79 mm            Acceptable
R0 = 20.0 kHz         Acceptable
R1 = 18.0 kHz         Acceptable
R2 = 12.4 kHz         Acceptable
R3 = 23.0 kHz         Acceptable
R4 = 30.7 kHz         Acceptable
T0 = 10.9 kHz         Acceptable
T1 = 18.3 kHz         Acceptable
T2 = 7.0 kHz          Acceptable
T3 = 14.5 kHz         Acceptable
T4 = 21.8 kHz         Acceptable
4. Simplified die design procedure

This slight design change has made the results fully acceptable, but note that the T4 frequency has reduced significantly. A further increase in SR would make the design unacceptable again, because T4 would be in the range 19-21 kHz.

This die was manufactured without the benefit of these design contour maps, and based on a single finite element analysis of the original design it was considered acceptable. After manufacture, the frequencies were found to agree fairly well with the predictions, as shown in table 4.01.

<table>
<thead>
<tr>
<th></th>
<th>PREDICTED FROM...</th>
<th>RESULT</th>
<th>ACTUAL MEASUREMENTS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>ORIGINAL ANALYSIS</td>
<td>CONTOUR MAPS</td>
<td></td>
</tr>
<tr>
<td>Outside diameter/mm</td>
<td>156</td>
<td>154</td>
<td>154</td>
</tr>
<tr>
<td>Frequency R0 /kHz</td>
<td>20.0</td>
<td>20.0</td>
<td>20.0</td>
</tr>
<tr>
<td>Frequency R1 /kHz</td>
<td>18.3</td>
<td>18.5</td>
<td>18.5</td>
</tr>
<tr>
<td>Frequency R3 /kHz</td>
<td>23.0</td>
<td>23.0</td>
<td>21.8</td>
</tr>
</tbody>
</table>

TABLE 4.01 COMPARISON OF PREDICTED AND ACTUAL RESULTS FOR 45 mm AEROSOL DIE

The contour map results agree very well with the original FE analysis which was specific to this die. Both sets of FE results agree well with the measured values from the actual die, except for the R3 frequency which at 21.8 kHz actual is significantly less than the 23.0 predicted. There are two likely reasons for this:

1) This type of FE analysis tends to overestimate resonant frequencies - for example an error of 3% would account for half the observed discrepancy.
2) The measured frequencies are invariably for the die with a transducer attached. The effect of the transducer (itself resonant at 20 kHz) is to pull the resonant frequency towards 20 kHz. Again, a 3% frequency change would account for half the observed discrepancy.

Note that for resonances below 20 kHz these errors will tend to cancel - and indeed the 18.3/18.5 kHz R1 frequency appears much more accurate. Therefore, special care should be taken to avoid harmonic resonant frequencies predicted above 20 kHz - say in the range 20 - 22 kHz.

4.3.2 Design of a Die for 211 Beverage Can Necking

Figure 4.08 shows a die for necking 66 mm diameter beverage cans to 58 mm diameter. For this design IR is 31 mm, SR may take any value from 40 mm upwards.
4. Simplified die design procedure

It is assumed that any combination of materials with a hard inner die is possible, and the predicted performance for all these combinations has been checked using the full set of design graphs in appendix 5. The results are shown in table 4.02, with unacceptable results marked '*'.

This shows that this design in any combination of materials could have problems with the R3 resonant frequency. For Al / EN41, Ti / Nk and Ti / EN41 the outside radius has also been marked unacceptable. This is because the bolster would be too thin to keep the pellet in a state of compression (or the bolster stress would be excessive). For these material combinations there is no advantage to be gained from increasing the value of SR - this if anything makes the bolster even thinner. Therefore, these combinations of materials can be eliminated from the design options.

<table>
<thead>
<tr>
<th>MATERIAL COMBINATION</th>
<th>R0</th>
<th>R1</th>
<th>R2</th>
<th>R3</th>
<th>R4</th>
<th>T0</th>
<th>T1</th>
<th>T2</th>
<th>T3</th>
<th>T4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Al/Sy</td>
<td>80</td>
<td>20.0</td>
<td>19.5*</td>
<td>21.0*</td>
<td>19.0</td>
<td>21.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Al/Nk</td>
<td>69</td>
<td>20.0</td>
<td>21.8*</td>
<td>20.0*</td>
<td>17.8</td>
<td>16.5</td>
<td>25.1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Al/EN41</td>
<td>60*</td>
<td>20.0</td>
<td>24.4</td>
<td>18.9*</td>
<td>15.5</td>
<td>18.1</td>
<td>18.1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ti/Sy</td>
<td>71</td>
<td>20.0</td>
<td>21.8*</td>
<td>21.5*</td>
<td>20.1*</td>
<td>16.0</td>
<td>24.1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ti/Nk</td>
<td>62*</td>
<td>20.0</td>
<td>23.9</td>
<td>20.1*</td>
<td>15.0</td>
<td>18.8</td>
<td>18.0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ti/EN41</td>
<td>55*</td>
<td>20.0</td>
<td>18.5*</td>
<td>16.5</td>
<td>18.5</td>
<td>19.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(Only frequencies in the range 15-25 kHz have been listed. Unacceptable results are marked *).

**TABLE 4.02 Predicted results for 66 mm die in various material combinations**
4. Simplified die design procedure

From the contour map of OR for Ti / Sy it was found that increasing SR also increases OR (eg, for SR = 56, OR = 87) and the bolster wall thickness OR-SR remains constant at 31 mm. This is not ideal - it should preferably be larger - but could be acceptable. Increasing SR in this way raises the problematical R3 frequency but also lowers the R1 frequency towards 20 kHz. No value of SR will provide acceptable frequencies for both R1 and R3, so this combination of materials can be eliminated.

From the contour map of OR for Al / Nk it was found that increasing SR for these materials does not give a corresponding increase in OR. For SR > 48 mm OR is approximately constant at 73 mm. This means that the maximum bolster wall thickness OR-SR is 29 mm for SR = 40 mm (barely acceptable). Furthermore, the R3 frequency for these materials cannot be increased above about 21.5 kHz. This combination of materials can, therefore, be eliminated.

The only remaining combination is Aluminium / Syalon, for which the design graphs are shown in figures 4.09 - 4.11. This combination gives a fully acceptable value for OR (80 mm giving a bolster wall thickness 40 mm) but unacceptable R1 and R3 frequencies. However, increasing SR to 50 mm would raise R3 to 22.0 kHz and lower R1 to 17.6 kHz. Checking the remaining frequencies reveals no other problems for this design, so a fully acceptable design has been derived: Aluminium / Syalon IR = 31 mm, SR = 50 mm, OR = 92 mm. Note that no other combination of available materials, and no lower value of SR, would give acceptable results.
4. Simplified die design procedure

Figure 4.09 OR, R0 & T0 contour plots for Syalon - Aluminium
4. Simplified die design procedure

Figure 4.10  R1, R2, R3 & R4 contour plots for Syalon - Aluminium
Figure 4.11 T1, T2, T3 & T4 contour plots for Syalon - Aluminium
SR HAS BEEN INCREASED TO 50 MM
FOR ACCEPTABLE MOUNTING ATTACHMENT THE LENGTH
OF THE INSERT HAS BEEN REDUCED BY 8 MM
MATCHING AREAS SO THAT AREA 1 = AREA 2 ... 
IR IS NOW 34 MM

FIGURE 4.12 'IDEAL' 66 MM BEVERAGE CAN DIE

Unfortunately, this design presents practical problems. The requirements for mounting ultrasonic dies are described in Chapter 5 and the preferred method is to use a resonant tubular mounting. The mounting dimensions are fixed by the sound velocity in the material and to minimise power loss, the material must be either titanium or aluminium alloy, both of which have very similar sound velocities. For either material the mounting is a tube with approximate dimensions inside diameter=82 mm, outside diameter=87 mm. The mounting must be joined rigidly to the back of the die (preferably it should be made integral with the die outer). This rigid connection is not practical for the ideal design with SR = 50 mm (ie, a 100 mm diameter pellet). The maximum diameter of pellet which was considered practical was 84 mm (SR = 42 mm). Figure 4.12 shows why this is so.

Returning to the contour maps, this design (Al / Sy with SR=42mm) is of course unacceptable with R1 = 18.7 kHz and R3 = 21.3 kHz. A new approach was needed. This die was manufactured with three equi-spaced flats on its
outside surface to modify the R1 and R3 frequencies. This type of "shaped die" is described in section 4.5.3 and in section 3.5.

<table>
<thead>
<tr>
<th>TABLE 4.03 COMPARISON OF PREDICTED AND ACTUAL RESULTS FOR 3-FLAT DIE</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Outside diameter/mm</strong></td>
</tr>
<tr>
<td>------------------------</td>
</tr>
<tr>
<td>R0 frequency/kHz</td>
</tr>
<tr>
<td>R1 frequency/kHz</td>
</tr>
<tr>
<td>R3 frequency/kHz</td>
</tr>
</tbody>
</table>

Table 4.03 gives a comparison of predicted resonant frequencies for a round die and a 3 flat design with actual measurements.

This shows the usefulness of the 3 flat design in modifying the R1 and R3 resonant frequencies. There is good agreement between the predicted (3 flat) and measured frequencies. The actual R3 frequency is again lower than predicted, for reasons described in section 4.3.1.
4.4 PROGRAMS

The contour maps described in previous sections represent the final result of many hours of computer analysis. In all, about 2500 different computer models have been analysed (11 combinations of materials, 6 values for inside radius, 13 values for shrink-fit radius and, on average, about 3 values for outside radius while optimising. \(11 \times 6 \times 13 \times 3 = 2574\)). To manually generate all these models, analyse them and collate the results would require far too much time and effort so the process has been automated.

All work is based on a simple parametric model, ie, parameters defined at the beginning of the program define the model to be analysed (explained in Chapter 2 Section 2.5). These parameters include the material for each part of the die (MATP - pellet material and MATB - bolster material, are numeric variables given numbers representing each possible material) and the die geometry (particularly IR - Inside Radius, SR - Shrink Fit Radius and OR - Outside Radius).

Initially the model is analysed for axisymmetric modes only, while the die is "tuned" by adjusting the value of OR until the resonant frequency in the RO mode is 20 kHz. A tolerance of +/- 0.2 kHz is allowed (+/- 1 per cent), but in practice the tuned frequency will usually be much closer than this because a small extra correction is made after the frequency is brought within the tolerance. When the OR value is finalised, the model is run again to predict all resonant frequencies up to the 4th harmonic (for explanation of harmonic numbers see Chapter 3 Section 3.1).

The procedure described above must be completed for every possible combination of geometry (IR and SR) and materials (MATP and MATB). This is made possible by another program which copies the original parametric program and modifies the copy to provide new parameter values. The user selects a range of values and a step size for each parameter and the submission program automatically produces a series of modified copies of the parametric program for each combination of the chosen parameters.
4. Simplified die design procedure

These parametric programs are run in series. Each completed program produces a set of results for frequencies and modeshapes and these results (together with the input parameters) are added to a results file. At this stage, the results are stored as text strings.

A command language program is then used to filter out the unwanted information on the results file and convert the results to integer format which can be conveniently read by a Fortran program.

Finally, the integer format results file is read by a Fortran program which identifies the modes from stored modeshape information. The mode and frequency information is then passed to a plot program which displays results as contour maps.

The functions of each program are described in more detail along with program listings in Appendix 3.
4.5 ENHANCEMENTS

This section describes further work which has extended the applicable range of the design procedure and other improvements which may be made as and when required.

4.5.1 New Material Combinations

At present, ultrasonic die designs are limited to eleven combinations of five materials. Direct replacement of the pellet materials with similar ones would be possible (eg, using tool steels other than EN41 or different grades of ferro-titanit or syalon). This should have little effect on the results shown on the contour maps provided that the material properties (particularly density and elastic modulus) are similar. Completely different materials which would serve as suitable alternatives have not yet been identified but new material property data can be added to the programs when available.

4.5.2 New Working Frequencies

The choice of 20 kHz as the working frequency is determined by the availability of proprietary ultrasonic equipment and by the requirement for maximum vibration amplitude. Achievable amplitude is limited by stress in the die materials (particularly the outer bolster) and the danger of fatigue failure. As frequency is increased, the wavelength (and hence all tuned dimensions) decreases in proportion. For a constant stress level the amplitude must be reduced in proportion. 20 kHz is the lowest commonly available frequency because at lower frequency the sound radiated is more audible to the human ear.

Equipment working at higher resonant frequencies is available, eg, at 22, 25, 30, 35 and 40 kHz. Although the amplitude must be reduced, one of these may be preferred. This would reduce audible noise and overall dimensions.
4. Simplified die design procedure

The design procedure described in this report is based on dies tuned to a working frequency of 20 kHz in the R0 mode. If any other working frequency is required this can be analysed by modifying the command program OPHTEST6.COM (see Section 4.4.3) to tune each die to the new frequency.

4.5.3 Shaped Dies

As described in Section 4.3.2, some die designs may not yield satisfactory results. In this case, some modifications to the basic round die have been designed which move the unwanted resonant frequencies in relation to the required R0 resonance. The use of these 'shaped' dies is described in section 3.5. Three suitable designs are shown in figure 4.13 together with their effects on the resonant frequencies of a solid steel die.

| Changes in Resonant Frequencies (Compared to Round Die) |
|----------------|----------------|----------------|
| R0 +0.3 kHz    | R0 +0.2 kHz    | R0 +0.3 kHz    |
| R1 -1.4 kHz    | R1 +2.1 kHz    | R1 -0.5 kHz    |
| R3 +0.6 kHz    | R3 +0.1 kHz    | R3 +1.3 kHz    |

Note: These frequency changes are for a solid steel die of 40 mm inside diameter and flats of 60° included angle. Smaller and larger flats will affect the frequencies correspondingly. Other dies will behave similarly but will not give identical results.

Figure 4.13  Typical Effects of 'Shaped' Dies
The frequency modifications provided by these shapes have been found to be fairly consistent over a variety of different die designs and material combinations. Therefore, it may be assumed that if the frequency behaviour of a proposed new die is unsatisfactory, it may be improved by using one of the shapes shown in figure 4.13. However for exact prediction of the resonant frequencies, it will be necessary to analyse the proposed die individually.

These shaped dies are covered by CaraudMetalbox UK and foreign patents [124].

### 4.5.4 High Radial Mode Dies

Figure 4.14 (bottom) shows the radial axisymmetric (R0) modeshape in which the ultrasonic dies are tuned to vibrate.
This is a satisfactory mode of vibration which provides a large uniform amplitude on the inside die surface and a convenient place to attach the transducer. However, there is another mode of vibration, also a radial axisymmetric mode, which would satisfy the conditions, as shown in figure 4.14 (top). There are indeed any number of these higher radial axisymmetric modes which would satisfy the conditions, but the one shown is probably the only one of interest. This mode leads to a die which is physically larger than a conventional ultrasonic die operating at the same frequency. It would be suitable for applications requiring a large inside diameter (e.g., 75 mm diameter or more) or for dies to operate at a frequency higher than 20 kHz.

If this design becomes commonly used, then similar contour maps for this mode could be prepared with slight modifications to the programs described in Section 4.4. Use of this mode would probably lead to additional unwanted modes of vibration so additional contour maps would be required, but this technique would be especially useful.
4.6 CONCLUSIONS

A new procedure has been devised to assist the design of ultrasonic dies for new applications.

The new procedure is based on a mass of data which has been generated by computer analysis of many different ultrasonic dies. Several materials and a range of possible dimensions have been considered. The data is displayed on a series of contour maps each of which shows the variation of one parameter against two variables. By careful study of these contour maps an engineer can design a new die with dimensions such that it is resonant in the R0 mode at 20 kHz. Furthermore, the resonant frequencies in other modes can be read from other contour maps to determine whether any other resonant frequency appears close to 20 kHz.
The design of ultrasonically vibrating tools described so far has concentrated on die design and analysis. This has led to a series of die designs based on the axisymmetric radial mode of vibration (R0). One of the features of this mode is that the whole die vibrates - there are no nodal points. This makes the design of a mounting system critical to the efficient operation of the die.

If a resonant system is fixed other than at a vibration node then there are three likely problems:

1) Energy will be lost from the vibrating system, limiting its efficiency.

2) Vibrations transmitted to the rest of the machine can cause various other problems e.g. loosening screws, damaging bearings etc.

3) The resonant frequency will change, again reducing efficiency

Figure 5.01 shows this diagrammatically.

To overcome these problems the die might be flexibly mounted but in a high speed production machine this would cause severe problems of tool alignment. The die would move under the influence of inertia forces or low-frequency vibrations and the resulting misalignment would cause jams, tool damage etc.

This chapter describes the work to find a solution to this problem. The work ultimately led to the geometric optimisation of a new tubular mounting system, covered by CarnaudMetalbox UK and foreign patents [125].
5.1 MOUNTING OPTIONS

From the preliminary discussion above it seems that the ideal mounting for ultrasonic dies is one which is both perfectly rigid (to maintain tooling alignment) and perfectly flexible (to avoid damping the vibrations). To reconcile these apparently conflicting requirements four methods of mounting the die have been considered, using a nodal flange, roller bearings, resonant rods or a resonant tube (figure 5.02).
5. Mounting design & optimisation

**Figure 5.02** - Comparison of different mounting systems

- Fixed on booster flange (optionally with radial rollers behind die)
- Fixed on resonant rods (3 or more around die)
- Fixed on resonant tube

Transducer flange might be used to mount assembly but normally it is only used to support a protective "can" guarding the transducer.

"Step up" booster is commonly used to increase amplitude at workpiece.

"1:1" booster is used because the amplitude required at the die is equal to the working amplitude of the transducer.

Force: 5 kN

Plastic welding system

Force: 0.5 kN

Metal forming system

**Figure 5.03** - Comparison of systems for plastic welding and metal forming
It is common practice in the plastic welding industry to mount the vibrating system (transducer, sonotrode and intermediate "booster" horn) on a nodal flange of the booster, or occasionally the transducer (figure 5.03). This is perfectly acceptable for plastic welding tools, where process forces are fairly low (typically up to 0.5 kN) and act axially on the tool. For a metal forming die the process forces are higher (about 5 kN for the necking of aerosol cans) and act along the die axis which is perpendicular to the transducer axis. This would cause potentially disastrous bending stresses in the booster. Furthermore the bending deflection of the booster would again cause tooling misalignment. A variant of this system called a "force insensitive mounting" was developed by Jones [23] for wire drawing - but again this was only intended to resist axial forces.

This method was therefore considered unsuitable for this application.

5.1.2 Roller bearing mounting

This type of mounting arrangement was used for some of the early work using ultrasonics at Wantage, and is similar to a system of linear roller bearings used by Smith, Young and Sansome [42], [43] in a rig simulating a deep drawing process. The die is mounted on a nodal flange on the transducer/booster unit as in figure 5.03. To avoid misalignment problems caused by bending strain the back face of the die is supported by a number of roller bearings (cam followers) arranged with their axes tangential to the die (figure 5.04).

The intention is that the rollers prevent axial movement of the die (and hence bending stress and strain) while permitting free movement in the radial direction. The problem is that using relatively high amplitude ultrasonics the radial acceleration of the surface of the die is extremely high (eg. 10 μm amplitude at 20 kHz gives 158 000 m/s²). Furthermore the movement of the die surface is not exactly radial, there is also an axial component caused by
the Poisson’s ratio of the die material. The movement is shown (exaggerated) in figure 5.04. This means that contact with the rollers must be intermittent and under these conditions the rollers will inevitably impact and skid on the back of the die. The consequence is that both the die and the rollers will rapidly wear.

This was found to be the case during trials of this mounting system on the prototype rig. Within a short time the contact surfaces of die and roller were badly pitted, even at a vibration amplitude only about half of that described above. For this reason this method of mounting was deemed wholly unsuitable for a production machine.

5.1.3 Resonant rod mounting

In this system of mounting (used by Biddell [38] and Kariyawasam et al. [36]) the die is supported by a number of rods fitted to the back face and aligned in the axial direction (figure 5.02). A similar system described by Biddell and
Sansome [37] uses a cylindrical mounting slotted to produce individual "beams" - similar to the rods.

The rods naturally offer a fairly stiff mounting (depending on the number and dimensions of the rods used) but are designed to resonate at the working frequency of the die so that they are effectively very flexible at this one frequency. This type of system has the potential to satisfy the seemingly impossible requirements of the application and combine rigidity with compliance.

The dimensions of the rods are clearly of paramount importance. The length and diameter must be chosen to ensure that one of the bending modes of the rod has a natural frequency corresponding to that of the die. This may be the fundamental natural frequency or any of the harmonics. Thus the length, diameter, harmonic number and material properties are all important factors.

The design of this type of mounting using finite element analysis is described in section 5.2.

Two major problems were encountered: the predicted power loss was excessive and fixing the rods to the die face was expected to be difficult. To overcome these problems a completely new type of mounting was devised.

5.1.4 Tubular type mounting

The tubular type mounting was designed to eliminate the problems of tuned rod mountings by providing a low-loss resonant system which could be fixed simply and rigidly to the die. The design proved to be extremely successful and has since been used for all new dies. The analysis and subsequent optimisation of this mounting is described in section 5.3.
The aim of this analysis is to investigate the design of this type of mounting and to select suitable dimensions for the rods. These must be sufficiently rigid to support the die and should be resonant at the working frequency (20 kHz) so as to permit free movement of the die.

One problem with this design is matching the movement of the free end of a rod to the movement of the die. Clearly all free bending modes of a rod involve rotation of the free end. When the rod is firmly fixed to the die the end will be forced to move radially without significant rotation. Therefore the design should be based on the natural frequency of a rod with a rotational end constraint, rather than a free rod. Figure 5.05 shows some natural modes and frequencies (predicted by Ansys) of a rod fixed at its centre with both ends free and with one end fixed to prevent rotation.

<table>
<thead>
<tr>
<th>BOTH ENDS FREE</th>
<th>NATURAL FREQUENCY / kHz</th>
<th>ONE END CONSTRAINED</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1.688</td>
<td>1.691</td>
</tr>
<tr>
<td></td>
<td>1.695</td>
<td>2.652</td>
</tr>
<tr>
<td></td>
<td>9.861</td>
<td>9.862</td>
</tr>
<tr>
<td></td>
<td>9.862</td>
<td>13.263</td>
</tr>
<tr>
<td></td>
<td>25.072</td>
<td>25.094</td>
</tr>
<tr>
<td></td>
<td>25.116</td>
<td>29.557</td>
</tr>
</tbody>
</table>

Figure 5.05 - Bending modes of rods with free ends and with end constraints
Furthermore (as discussed previously) there is associated with the radial motion of the die a significant axial motion. The end of the rod will be forced to follow this also. In order to allow for this movement also the rod must also be made resonant in an axial mode. These design constraints are shown in figure 5.06.

With this basis the design becomes more closely defined. If the first axial mode is used then the simplest rod will be a half wavelength long (typically approx. 130 mm at 20 kHz) with a central flange for mounting. The diameter may then be chosen to tune the bending mode (with rotational end constraint) to the same frequency.

For an aluminium rod 17.5 mm diameter, 128.5 mm long fixed at the centre Ansys predicts natural frequencies of 19.977 kHz (axial) and 20.088 kHz (bending with end constraint) as shown in figures 5.07 and 5.08.
5. Mounting design & optimisation

FIGURE 5.07 AXIAL MODE OF ALUMINIUM ROD 17.5 DIA X 128.5 LONG

FIGURE 5.08 BENDING MODE OF ALUMINIUM ROD 17.5 DIA X 128.5 LONG
This appears to meet the requirements for the die mounting. To test the design further a more detailed finite element model is required. This must either include the die or (more simply) simulate the exact motion of the free die and apply this to the tip of the mounting rod. The aim is to predict how much force must be applied to the rod in order to cause the prescribed motion (and hence to determine the reaction force which will be applied to the die).

The evaluation of forces applied to a vibrating system at any given frequency requires a harmonic response analysis (see section 2.2.3). Given a force or displacement input the corresponding reaction forces and displacements can be calculated at a series of frequencies in a specified range. If the input is a constant force then the displacement output will be a typical resonance peak. If a constant displacement is applied then the reaction forces show the inverse i.e. a minimum at resonance. Figure 5.09 shows the form of these resonance curves.
From the applied displacement and the calculated force it is possible to calculate the power input, as follows:

The forces and displacements are varying sinusoidally; amplitudes (peak values) are used in the calculations. Power is usually calculated as the 'dot product' of Force and Velocity, which can be expressed as the velocity multiplied by the component of force which is in phase with it. The magnitude of velocity may be calculated as the angular frequency multiplied by the amplitude, and it is 90° out of phase with the amplitude. Hence the power input can be calculated by multiplying the applied amplitude, the component of force at 90° phase angle to it and the angular frequency.

At resonance this instantaneous power takes the form of a sine squared curve. The above calculation gives the peak value, so the average value is half of that calculated. Note that for axisymmetric analysis the Ansys program expresses forces on a 'per radian' basis. The total force is calculated by multiplying by $2\pi$. These calculations were set up in the Ansys command file using parameters as calculation variables. Power input was then calculated automatically for each frequency so that the variation with frequency could be studied.

Figures 5.10 to 5.12 show the results of this type of analysis. In figure 5.10 a fairly typical resonance curve is shown. This is the response of the rod to a constant force excitation. For this work the response to a constant amplitude excitation is of more interest. Figure 5.11 shows the radial force required to maintain the specified movement. Note how broad the resonance curve is - the force only varies from 120 to 150 N over a range of 4.5 kHz. At resonance (just above 20 kHz - in line with the earlier models) the force is still high.
5. Mounting design & optimisation

**Figure 5.10** Amplitude response of aluminium rod

**Figure 5.11** Force response of aluminium rod
Referring to the power curve shown in figure 5.12, this was calculated as described above to include the average power loss per rod. The minimum of the power curve appears at a lower frequency (approx. 19.5 kHz) because this is obtained by multiplying the force by the frequency, so the power is lower at lower frequency. Once again the curve is very broad. The minimum power value (about 240 W) is unacceptable for a high efficiency system because at least three rods will be required to fix the die. A typical ultrasonic generator produces only about 2 kW so this mounting system would waste about a third of the available power.

This type of mounting system suffers from another problem also. The results from the model used for figures 5.11 and 5.12 also indicated that 20 Nm torque was required to prevent the end of the rod from rotating. This level of torque (alternating in direction at 20 kHz) is liable to shake loose any mounting system used to fix the rod to the die. Furthermore the rigidity of the mounting system could drastically affect the resonant frequency (see for
example the difference between the natural frequencies of the free end and fixed end rods of figure 5.05).

One possible solution would be to use a thinner rod, resonant in a higher mode. A thin rod, which would bend more easily, would reduce both the torque at the connection to the die and the power consumption but there would be a danger of it buckling under the axial load applied to the die during can forming. It was consideration of these problems which led to the development of a completely new tubular mounting system.
5.3 TUBULAR MOUNTING

5.3.1 Initial tubular mounting design

It is desirable for the mounting to bend easily in the radial direction but still be resistant to buckling. To improve on the buckling strength of a thin rod a thin walled tube was considered. This derives strength from its curvature. For example compare a tube with a rod of equal thickness:

Euler's formula predicts that a rod, simply supported at both ends, of rectangular cross section 2.5 x 25 mm, 60 mm long, made from aluminium (Young's Modulus 74 GPa) will buckle under a load of 6.6 kN.

Applying the formula given by Roark [3] to the buckling of a tube of 85 mm average diameter, wall thickness 2.5 mm and length 60 mm the predicted buckling load is 880 kN. This is far greater than would be achievable using a number of the corresponding rods (and in fact is far in excess of the yield strength of this cylinder).

These calculations are given in full in appendix 6.

Naturally the curvature of the tube affects not only the buckling strength but also the bending resonance. If the tube is to distort axisymmetrically then it must expand and contract in diameter. In fact for a thin walled tube bending stiffness is largely replaced by hoop stiffness.

A feature which has been known for many years in the design of tuned ultrasonic dies (eg Sansome and Biddell) is that as the inside diameter is increased the outside diameter must be reduced to maintain the same natural frequency. Taking this to its limit gives a die with a very large bore and a small outside diameter ie. a thin tube. This suggests that a bending resonance should not be required - the radial resonance of the tube should allow the mounting to match the radial movement of the die.

The initial tubular mounting design was based on this theory (figure 5.13). It is a thin walled tube of a diameter such that radial resonance is achieved and
of a length to match the axial mode half-wavelength. To fix the mounting to the machine a central flange is fitted, which will be nodal in the case of the axial mode. Note that the flange will not be naturally nodal for the radial mode but the flange will itself detune the radial mode at that point and so should reduce the amplitude of vibration there.

Analysis of this mounting alone, with both ends free, showed a multitude of vibration modes at frequencies close to the target 20 kHz. The reason for this is that the natural frequency is largely determined by the tube diameter (i.e. by hoop stresses) but it is also affected by the length of the mounting and the modeshape (by bending stresses). The tube diameter dictates radial resonance at about 20 kHz and the numerous modes each modify this frequency to some extent. Figures 5.14, 5.15 and 5.16 show modes at 19.781, 20.366 and 23.247 kHz respectively.
5. Mounting design & optimisation

![Figure 5.14 Initial Tubular Mounting - FE Results for First Mode](image1)

![Figure 5.15 Initial Tubular Mounting - FE Results for Second Mode](image2)
5. Mounting design & optimisation

**FIGURE 5.16 INITIAL TUBULAR MOUNTING - FE RESULTS FOR THIRD MODE**

**FIGURE 5.17 NATURAL MODE OF TUBULAR MOUNTING WITH TIP CONSTRAINT**
If the end rotation of the tube is constrained (as for the rod design) then there are fewer modes in the 20 kHz region. This is a better representation of the true situation. Figure 5.17 shows an axisymmetric mode at 20.251 kHz with the tube tip constrained against rotation.

For a tubular mounting fixed in the centre with one end fixed to the die and the other end free some of the modes of vibration are associated almost entirely with the motion of the free 'tail', while others involve mainly the fixed end. All of the modes shown in figures 5.14 to 5.17 will therefore appear.

This suggests that the mounting could work without the tail, in complete contrast to the simple theory which requires the mounting to be symmetrical about the flange in order to support the axial mode of vibration. Although the mounting tail does appear to improve the overall performance of the mounting (see section 5.3.4) the analysis is greatly simplified by ignoring it at this stage. As the following results show, analysing the mounting without its tail allows its behaviour to be much more easily understood.
5. Mounting design & optimisation

The power input to the mounting can be calculated as before, using the force required to maintain vibration at a given amplitude. As for the tuned rod the power input can be calculated from the force and amplitude. The graph of power input against frequency again shows the characteristic minimum at a resonant frequency. Figure 5.18 shows this for the initial design. The resonant frequency in this case is approx. 20.9 kHz and the minimum power input is about 20 watts. Note also that at 20 kHz (the nominal working frequency) the power input is approx. 500 watts.

5.3.2 Optimising tubular design

The basic principles of the tubular mounting (simultaneous resonance in the radial and axial modes) have led to a usable mounting design but one which is evidently not the ideal. Other designs might offer a power curve which is better centred on the working frequency (20 kHz), which uses less power at minimum or which offers a wider spread of low power operation. Variables in the mounting design which might be changed include material, wall thickness, radius and length. In practice the choice of materials is limited as for the die outer (see section 3.6) and the wall thickness is dictated by the required strength (a minimum thickness is desirable for minimum power loss). This leaves the diameter and length as variables which can be used to optimise performance.

Optimisation facilities are available in Ansys but with only two variables there is another option available to the user. This is to evaluate the performance for a range of both variables and display the results on a contour graph. This has the advantage of providing an insight into the behaviour of the mounting. Using a parametric model of the mounting (section 2.6) a series of analyses were run for different values of diameter and length. The results were collated in contour maps of resonant frequency (defined as the frequency of minimum power loss) and power at 20 kHz. These were produced for mountings made from aluminium and titanium alloys with a 2.5 mm wall thickness.

Figures 5.19 to 5.22 show the results. For each material two contour graphs have been produced, for the resonant frequency and the power consumption at 20 kHz.
Figure 5.19 Contour plot of resonant frequency for aluminium mounting

Figure 5.20 Contour plot of power at 20 kHz for aluminium mounting
Figure 5.21 Contour plot of resonant frequency for titanium mounting

Figure 5.22 Contour plot of power at 20 kHz for titanium mounting
The shape of the contour plots shows the ideal geometry for the mounting. The 20 kHz contour on the resonant frequency plot shows a range of possible combinations of radius and length. These naturally correspond to the low power areas on the 20 kHz power contour plot but note that some of these combinations are much better than others. For aluminium (figures 5.19 and 5.20) in the area around radius 41.5 to 42.0 mm, length 60 to 70 mm the power is generally low and the resonant frequency varying slowly, whereas in other areas (eg radius 43.0 mm, length 50 mm) the power and resonant frequency change much more rapidly. For this reason the geometry initially selected for the mounting was radius 41.7 mm, length 65 mm. This ensures that slight errors in geometry (eg from machining tolerances) will have minimum effect on the die performance and also gives maximum allowance for other sources of error (eg material properties, die natural frequency, analysis inaccuracy etc.).

Comparing the corresponding graphs for titanium (figures 5.21 and 5.22), the results are broadly similar. The ideal geometry in this case is radius 40.7 mm, length 65 mm. The similarity is to be expected because the sound velocity in the two materials is almost the same (Titanium 5130 m/s, Aluminium 5140 m/s using the figures given in appendix 4). This means that almost exactly the same geometry can be used for either material - the choice of materials depending on the properties required. Section 3.6 describes the material selection in detail (the requirements of the mounting are the same as for the die outer). Titanium will offer the best strength, toughness and fatigue resistance while aluminium offers lower cost and minimum power loss.

5.3.3 Maximum length

Note that for lengths greater than about 80 mm the low power area disappears altogether although the frequency plot continues to indicate a 20 kHz resonance. This is a result of the mounting beginning to act independently, vibrating at a much higher amplitude than the die.
In this application it is obviously undesirable for the mounting to behave in this way, and it is therefore very important to understand the reasons for its change of behaviour. As long as the mounting tube is short it does not have the freedom to move independently of the main system - the amplitude of the mounting is dictated by the amplitude of the die, and the mounting works as intended. If a longer mounting tube is used then it does have the freedom to vibrate independently and so can vibrate at high amplitude, absorbing energy from the rest of the system.

The contour maps of figures 5.20, 5.22 show clearly that the mounting design should be limited to a maximum of approx. 75 mm long in order to avoid this problem.

5.3.4 Tubular mounting tail design

The optimisation of the mounting was based on a simple tube joining the die to a fixed flange, but the original design also featured a 'tail' section of tube behind the flange. This was not used in the optimisation process because the additional resonances of the tail section significantly complicated the results, and would have made the selection of an optimum design much more difficult. Furthermore in the resonance modes which were of interest (i.e. those which involved relatively large motion of the die) the tail section was essentially stationary. This led to the belief that the mounting should work adequately without the tail, and this was shown in the above results.

After an optimum design was derived the use of a tail section was investigated again to determine whether the performance of the mounting could be further improved. Power losses due to the mounting are largely caused by energy transfer through the flange into the supporting structure. To minimise these losses it is necessary to reduce the vibrations in the flange. The mounting tail has the potential to achieve this by acting as a vibration absorber.
This type of behaviour is well known and described in standard texts on vibration (Den Hartog [111], Timoshenko[112]). Typically a vibration absorber is added to an existing system in order to reduce its amplitude of vibration. A simple spring-mass pair tuned to the frequency to be suppressed is fitted to the vibrating system. This creates an extra degree of freedom and has the effect of shifting and flattening the resonance peak (see diagrams given in the above references).

The tail section of the mounting was assumed to be the same diameter and wall thickness as the main mounting tube, and its length was variable - to be determined. A parametric analysis similar to the previous one was performed, and the results displayed as a function of the tail length.

In this case the power loss was not the principal concern. Any design requiring less than 100 W to run would be considered acceptable. Instead the main focus of the optimisation in this case was improving the tolerance of the mounting, i.e. extending the range of the low power area. This was done by studying the resonance curves for designs of different tail lengths. The aim was to achieve a wide, flat curve such that the power loss would be less than 100 W over a large frequency range.

Since the requirements of the tail are broadly similar to those of the rest of the mounting tube (i.e. radial resonance at the working frequency), it was natural to design the tail as in the original concept i.e. a tube of the same inside and outside diameters as the mounting, of length to be determined. However the optimum shape for the tail was not known, so two other shapes were also tried to determine whether any further advantage could be gained.
Figure 5.23 shows the three different tail shapes tested.

As before, the results were displayed as a contour map to give the user an indication of the range of behaviour of the mounting. In this case the contours display power as a function of frequency and tail length. Hence each resonance curve would be a horizontal section through the contoured surface, and because this is a power resonance curve there is a minimum at the resonant frequency.
5. Mounting design & optimisation

Figure 5.24 shows the contour map for the 'standard' tail on a titanium mounting. At the base of the contour map (for tail length zero) the results correspond to the resonance curve for the optimised mounting without a tail. It can be seen that as tail length increases so the width of the resonance curve also generally increases, although very irregularly. For some values of tail length (eg 70 mm) two or more resonances are apparent, indicating that the mounting would operate at frequencies other than the designed 20 kHz. It was to avoid this type of complex resonance curve that the mounting tube was originally optimised without the tail.

Note that the mounting tail is acting as a complex vibration absorber - widening the resonance curve and dividing it into a number of resonances. In contrast to the situation described in the previous section, however, in this case the behaviour is beneficial. Here vibration energy is absorbed not from the die but from the mounting flange, which is intended to be stationary.
For the purpose of optimising the geometry it is desirable to choose a tail length which widens the resonance curve as far as possible. Two areas are highlighted on the contour plot of figure 5.24 - lengths 50 and 80 mm, where the range of sub-100 W bandwidths are 500 and 730 Hz respectively compared to 300 Hz for the basic mounting without tail. In practice either one could be chosen according to the amount of space available, the required cost or other considerations. Alternatively the mounting could be designed without a tail, although it will be less tolerant to frequency variation, manufacturing errors etc.

The contour plot of figure 5.25 shows the results for the thickened tail (option 2 in figure 5.23). Again the best tail lengths are 50 and 80 mm but here the sub-100 W bandwidths are 690 and 900 Hz respectively.
Probably the best result is obtained using the tapered tail (option 3 of figure 5.23), as shown in the contour plot of figure 5.26. For this design the 50 mm tail length gives a bandwidth of 830 Hz (results for longer tails do not show any advantages).

These results show that using the 'standard' tail (at a convenient 50 mm long) the frequency range over which the mounting will operate efficiently can be increased by 70 % (compared to the mounting without tail). Using the tapered tail design the usable frequency range can be increased by 175 %.

The results shown are for a titanium alloy mounting, but similar results apply to the aluminium mounting also.

Alternative geometries could no doubt be designed which would further improve the performance of the mounting, but at this stage perfectly adequate performance has been achieved and the design work was stopped here. Most
tubular mountings were in fact built to the previous version of the design (i.e. standard tail).

5.3.5 Tubular mounting- finalised design

From the work described above the following dimensions (in mm) were specified for tubular mountings:

<table>
<thead>
<tr>
<th>Material</th>
<th>Inside radius</th>
<th>Outside radius</th>
<th>Length from die to flange</th>
<th>Length of tail</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aluminium L168</td>
<td>41.5</td>
<td>44.0</td>
<td>65.0</td>
<td>50.0</td>
</tr>
<tr>
<td>Titanium Ti64</td>
<td>41.0</td>
<td>43.5</td>
<td>60.0</td>
<td>50.0</td>
</tr>
</tbody>
</table>
5.4 SUMMARY

Three existing systems for mounting ultrasonic dies were considered: nodal flange, radial rollers and tuned rods. Each had severe shortcomings for the application under consideration. Therefore a fourth type of mounting was devised: a thin walled tube. Simple analysis gave rise to an initial design which was then optimised by finite element analysis of similar mountings over a range of frequency and dimensional geometry. The result is a mounting design offering low power loss and tolerant of errors and frequency variations. The design is covered under CarnaudMetalbox UK patent [125].
The die design work described in the previous chapters has been largely theoretical. This was backed up by practical verification at the time, to ensure the validity of the theoretical models. Some of this verification work was by comparison with other theoretical models, and this was described in chapter 3. The work described here is the ultimate verification - the physical testing of dies manufactured for various processes during the course of the project.

For the ultrasonic dies and mountings designed by finite element analysis the essential verification involved simple checks on the performance and the longevity of a component. More detailed testing was also used to help estimate the likely accuracy of future finite element models.

The purpose of this chapter is to describe methods of testing the ultrasonic equipment, to record the measurements made and to compare the measurements where possible with the theoretical expectations.
6.1 EVALUATION OF MODE AND FREQUENCY

Electronic equipment for the measurement of frequency is readily available. This can be used to measure the driving frequency of the power supply to the die, but to be meaningful this information must be related to the response of the die.

The simplest method of evaluating vibrations is using a power meter to monitor the electrical power being supplied to the transducer. Provided that the high frequency power supply operates at (approximately) constant voltage the input power will peak at resonance. Voltage is effectively equivalent to force, and current to amplitude. This is a useful method of finding a resonance but gives no information on the mode of vibration.

There are several 'traditional' physical methods for determining the die's motion as illustrated in figure 6.01. The relevant ones will be described in turn.

![](image)

**FIGURE 6.01 - "TRADITIONAL" METHODS OF MEASURING DIE VIBRATIONS**
6.1.1 Physical methods for evaluating mode of vibration

One simple and sensitive method of testing the vibrations is the 'feel' of the die surface - activity can be gauged roughly by gently touching the vibrating surface. High amplitude vibration is indicated by an oily, frictionless feel. Note that this technique is not generally recommended because of possible medical effects which may include tissue damage due to vibrations and burns from energy transfer. Ultrasonics are used in physiotherapy but at higher frequency and much lower power. Some of the physical/safety effects of ultrasonics are discussed in section 1.4.

Another physical output of most ultrasonic systems which also has obvious safety implications is the airborne noise. Using a sound level meter at a fixed distance this can be used as a measure of die activity. By moving the meter around the die at a fixed distance it is possible, in theory, to find areas of high and low activity. In practice, however, picking out different modes of vibration by this method is very difficult because of the radiation of noise in all directions from all parts of the die.

Use of a sound level meter will also show whether the operator is at risk from the noise emitted - hearing protection should be worn if the operator is to be close to unshielded equipment for a significant time. Again, see section 1.4 for a discussion of the safety aspects of ultrasonic noise.

A simple, non-hazardous method of indicating the mode of vibration is using fine powder (talc). The die must be set up with its axis vertical and while it is vibrating in some unknown mode the talc is sprinkled on the flat top face. The vibrations cause the talc to move over the surface and each mode of vibration shows a characteristic pattern of talc flow. The use of this technique to evaluate vibrations of circular plates (producing "Chladni figures") was described by Waller [113]. For example, if the die was working properly in the R0 mode (as described in chapter 3) then the talc would flow smoothly inwards to the hole in the centre, whereas if the die was vibrating in the R3 mode there would be six areas (corresponding to the nodal points) where the
talc would not move or would move tangentially. Figure 6.02 shows some talc patterns observed, and these are discussed in section 6.2.1.

A more sophisticated method used for measuring the amplitude on the outside surface of the die was the Telasonic amplitude meter. This device uses an eddy-current sensor for sensitive measurement of distance and electronic processing to produce a digital readout of amplitude perpendicular to the surface. No special mounting is required - the probe is simply held close to the vibrating surface. The measurements depend on the electrical properties of the material so the system is only applicable to aluminium and titanium (the materials for which it was calibrated). Because of its high purchase price this piece of equipment was obtained only on hire to evaluate the motion of a specific die when required. See the work of Chapman and Lucas [114, 115] for some results obtained from this equipment and used in modal analysis.

![Figure 6.02 - Patterns of Talc Movement](image-url)
All of the methods described above were used in early work on ultrasonics at CarnaudMetalbox. The high frequency power supply used at that time offered full manual control of frequency. This allowed the user to drive the ultrasonic tooling over a wide range of frequencies and determine its response, using a power meter and by physical methods. For later work, however, ultrasonic systems from the plastic welding industry were used. The single-frequency dedicated ultrasonic generator and transducer were capable of operating at much higher efficiency than the earlier system (and hence providing greater amplitude) but were not suitable for operating over a wide frequency range. Furthermore the operating frequency is not under the control of the operator, instead an automatic frequency control 'latches on' to resonance. These systems are therefore not suitable for full testing of the ultrasonic tooling. Appendix 7 contains specifications for the ultrasonic generators and transducers used in this work.

6.1.2 Admittance / Impedance Plotter

Following the adoption of single-frequency ultrasonic systems there was a need for a dedicated test system to perform the measuring functions which the new equipment could not handle. The equipment selected (called an Admittance / Impedance Plotter and supplied by Sonic Systems [116]) comprises a variable frequency generator with output voltage suitable for piezo-electric transducers. A digital frequency meter gives an accurate measure of the output frequency, which can be adjusted manually or set to scan automatically over a range at variable scan rates (for specifications see Appendix 7).
FIGURE 6.03 - MEASUREMENTS USING "ADMITTANCE PLOTTER"
The most useful feature however is the signal output of the electrical admittance (or impedance) of the transducer. Two signals are provided, corresponding to the real and imaginary parts of the admittance / impedance and these should be connected to the X and Y inputs of a chart recorder. Thus the complex admittance or impedance is displayed as a point on a polar plot (a conventional form of display for complex numbers) and the variation with frequency is displayed as a curve.

This is useful because over a wide frequency range (e.g., 16 to 24 kHz) the electrical properties of the ultrasonic transducer change only gradually. The observed variation in electrical admittance or impedance depends almost entirely on the mechanical resonances of the transducer and anything attached to it. As the frequency is scanned through a resonance the admittance plot describes a circle, while scanning through an antiresonance causes the impedance plot to describe a circle.

Of these, the admittance plot is generally the more useful. It can be shown [68], [116] that the point of zero imaginary admittance corresponds to resonance while the points of minimum and maximum imaginary admittance (at 90° on the circle) correspond to the half-power points. Furthermore the size of the circle is inversely proportional to the power losses in the system. (Note however that this information should be treated with caution - this is an indication of power loss at low amplitude but material damping can be very non-linear so results at high amplitude may not correspond.) Figure 6.03 shows the general arrangement of equipment and a typical circle plot with the significant features marked for clarity.

By careful study of the admittance plot and the frequency meter the following information can be obtained for each resonance:

Resonant frequency
Half power bandwidth
'Q' factor
System losses
With a little further effort the user can also gain a clear idea of the mode of vibration. The generator frequency should be set to resonance with the pen plotter (or a simple voltmeter) indicating the real component of admittance. When the die is touched the extra losses introduced by a human finger are detectable by a reduction in admittance (the circle becomes smaller). The safety concerns about touching vibrating tools described in the previous section are not relevant here because the power output of this generator is very much smaller. The reduction in admittance depends on the mode of vibration and where the die is touched. Touching at a vibration node will have little or no effect on the admittance while touching at an antinode will have most effect. Thus by touching the die and watching the admittance display the user can quickly identify nodal areas (if any) and hence establish the mode of vibration. Figure 6.04 demonstrates the principles of this method.
6.2 RESULTS OF MEASUREMENTS ON ULTRASONIC DIES

6.2.1 Mode evaluation using talc

Figure 6.02 shows some typical talc patterns found in early work on this project. In these diagrams the arrows show the direction of the observed talc flow and the relative size of the arrows indicates roughly the speed of flow.

Note that while the relative amplitude of different areas of the die is clearly shown (by the size of the arrows) all phase information is lost. In both the radial axisymmetric (R0) and the radial first harmonic (R1) mode the areas at the top and bottom of the die (in this view) appear identical. The arrows indicate that for both modes the talc moves quickly towards the centre of the die. The important difference is that in the R0 mode these areas are in phase, moving inwards and outwards together, whereas in the R1 mode they are 180° out of phase ie. the top moves inwards while the bottom moves outwards. (See section 3.1 for a detailed description of these modes.) It is this phase difference which accounts for the cancellation of amplitude over a part of the die which is seen in the combination modes. Depending on the phase angle between the R0 and R1 modes and their relative amplitude, cancellation of movement can occur at either top or bottom of the die or anywhere in between, while in other areas the movement due to the two modes will add to give increased motion.

At the time these measurements were made the unwanted harmonic modes of vibration were not understood and the uneven modes of vibration were blamed (wrongly) on a poor interference fit between the inner and outer parts of the die.

6.2.2 Admittance plotter measurements

In later work finite element analysis was used to help in understanding the unwanted modes of vibration and in predicting their natural frequencies. The
admittance plotter measurements were so complete and accurate that most
work in evaluating the performance of ultrasonic dies was done using this
method. Tables 6.01 to 6.05 and 6.12 to 6.13 show many of the results
obtained. In each case the resonant frequency, half-power bandwidth and
circle diameter (in mV) is recorded for each resonance, along with any other
relevant measurements obtained (eg. power input at a given amplitude level).
Note that power and amplitude measurements must be treated as
approximate indications because of unknown calibration of the various
meters, although comparisons should be valid between measurements using
the same ultrasonic equipment.

The measurements taken, and the corresponding descriptions / comments
have been divided into four categories: transducers, die tuning and
miscellaneous dies and mountings are included here. Use of the admittance
plotter in evaluating the mounting is described in section 6.4.2.

6.2.2.1 Admittance plotter measurements - transducers

Table 6.01 shows measurements of various transducer systems used for this
work (see appendix 7 for equipment details).

Items 1 to 7 are all measurements of the same component (the Kerry
Ultrasonics transducer serial number 757). Multiple measurements were
made because a circle plot of the transducer alone was usually made as a
reference at the beginning of each series of tests (eg tuning a die). This
shows some variability in the measurements. The transducer varies in
frequency from 19911 to 20035 Hz and in circle diameter from 56 to 63 mV.
The measurements were made with the transducer resting on a flat surface
(eg a table or bench) so the changes in circle diameter (indicating changes in
power losses from the component) could be caused by differences in its
positioning or in the coefficient of friction on the surface. This would not
account for the changes in resonant frequency, however. Some variability
would be expected as a result of temperature variations but there is also a general trend of frequency rising with age. This might be caused by gradual aging of the piezo-ceramic crystals or by "bedding in" of the interfaces. In any case the rate of frequency rise is not great enough to cause any problems.

Items 8 and 9 are measurements of the second Kerry transducer, serial number 786, which was apparently identical to the first. These show a slightly higher resonant frequency and a smaller circle diameter (average 51 mV compared to 60 mV). This indicates a slightly greater power loss from the second transducer. The greater half-power bandwidth of transducer 786 (34 Hz compared to 27 Hz average) also indicates higher power losses. Note that the half-power bandwidth can only be used to compare power losses in this case because the two components are nominally identical. At constant frequency the power loss is proportional to half-power bandwidth and to stored energy. The stored energy is a function of size, shape and material so it should be the same for two identical transducers.

Item 10 is another transducer from a different manufacturer (Telsonic Ultrasonics) which also features an integral "can" to guard the electrical connections to the transducer (useful because high voltages are used) and also to protect the transducer from damage due to accidental knocks. Despite this extra component which must absorb some power, the large circle diameter indicates lower power losses than for either of the other transducers. In fact this transducer is made from aluminium alloy, whereas the others are made from titanium alloy and steel. The moving mass of the Telsonic transducer is therefore less than that of the Kerry transducers and hence its stored energy is less. It is this that allows the Telsonic transducer to operate with lower power losses. The difference in stored energy could also be deduced from consideration of the half-power bandwidth of the Telsonic transducer - it has a larger half-power bandwidth (49 Hz compared to 34 or 27 Hz) but a smaller power loss (indicated by the circle diameter) so its stored energy must be less.
<table>
<thead>
<tr>
<th>ULTRASONIC EQUIPMENT TESTED</th>
<th>DATE</th>
<th>DIE DIA (mm)</th>
<th>TUNING STATUS (mm)</th>
<th>FREQUENCIES /Hz</th>
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<th>RES FREQ</th>
<th>BANDWIDTH</th>
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<td>-</td>
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<td>19705</td>
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<td>59</td>
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<td>-</td>
<td>-</td>
<td>19893</td>
<td>44</td>
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<td>19894</td>
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<td>-</td>
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TABLE 6.01 - ADMITTANCE PLOTTER MEASUREMENTS - TRANSDUCERS AND BOOSTERS
Item 12 shows the result of fixing the Kerry transducer (number 786) to the magnetostrictive transducer system used for the early work on this project. This result is remarkable for the huge half-power bandwidth and the tiny circle diameter which indicates very high power losses. The inefficiency of the magnetostrictive system in comparison with the piezo-electric ones has also been demonstrated by direct measurement of power and amplitude so this type of result is to be expected. Note also that the large half-power bandwidth has a positive benefit in allowing the transducer to operate over a wide frequency range, as discussed in section 1.1.5.

Items 14 to 21 show the results of fitting various interstage horns ("boosters") to the Kerry transducers. The aim was usually to give a reduction in amplitude because the transducer was vibrating at approx 27 μ whereas the die design indicated a limit of 20 μ. Measurements with the booster alone (items 14,16,21) show a slight reduction in circle diameter, indicating a small increase in the power loss. The other reason for using a booster was to fit a guard (case) around the transducer. This was fixed to a nodal flange on the booster so that (theoretically) no energy would be transmitted from the vibrating system to the case. In practise the measurements indicate a further increase in the power loss and the system with booster and case had a circle diameter about half that of the transducer alone (ie twice the power loss). Measurements were also made of the power and amplitude (using the meters on the Kerry system). Reference to these and the circle diameters suggests that approx 150 W is required to operate the Kerry transducer at 27+ μ, with a further 150 W lost in the case, if fitted.

One further comparison is also shown in this series of measurements (items 17 and 18). A system comprising transducer, booster and case was measured with the Admittance plotter connected directly to the transducer, and connected via a cable 3 m long (this cable was used to supply the transducer while it was used for can necking in the hydraulic press). The direct / indirect switch on the admittance plotter was also changed
accordingly. There was no significant difference between the readings, indicating that the use of the extra cable did not impair the performance of the ultrasonics.

6.2.2.2 Admittance plotter measurements - die tuning

Tables 6.02 to 6.04 show the tuning procedure for six ultrasonic dies. The method of tuning a new die is described in detail in section 3.8, but the essential point is that the die must be manufactured oversize on outside diameter and gradually machined down until the resonant frequency meets the requirement (usually 20 + 0.1 kHz).

Items 1 to 7 in table 6.02 show the tuning of a die constructed from a titanium alloy outer with a ferro-titanit insert, used for necking a 45 mm diameter aerosol can to 33 mm diameter. Note that for this system (die and transducer) three resonances have been found at each stage of the tuning process. By widening the frequency range many more resonances could have been found for all systems tested. In general only those in the range 18 to 22 kHz have been noted because outside this range the other resonances have not been found to cause a problem.

For the initial measurement of the die as manufactured resonant frequencies were found at 18.1, 19.6 and 21.5 kHz. From the finite element analysis it was expected that the R1 (first harmonic) would appear below the R0 (axisymmetric) and the R3 (third harmonic) above it. It was therefore expected that the three resonant frequencies corresponded to R1, R0 and R3 modes respectively. Touching the die and noting the response of the admittance plotter (as described in section 6.1.2) confirmed this. In general the resonant frequencies have been recorded with the R0 frequency first, followed by the unwanted resonances.
<table>
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<th>DATE (mm)</th>
<th>DIE DIA (mm)</th>
<th>TUNING STATUS (mm)</th>
<th>FREQ RES</th>
<th>BANDWIDTH</th>
<th>CIRCLE DIA</th>
<th>RES FREQUENCIES</th>
<th>BANDWIDTH</th>
<th>CIRCLE DIA</th>
<th>ANTIRES FREQUENCIES</th>
<th>BANDWIDTH</th>
<th>CIRCLE DIA</th>
<th>ANTIRES FREQ</th>
<th>COMMENTS / OTHER MEASUREMENTS</th>
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**TABLE 6.02 - ADMITTANCE PLOTTER MEASUREMENTS - DIE TUNING (1 OF 3)**
As manufactured the R0 frequency was too low. This is normal because the
die is made oversize with an allowance for tuning. During tuning the outside
diameter of the die was progressively machined down, in this case from 156
mm diameter to 154, 153 and finally 152 mm. As a result the R0 frequency
increased towards 20 kHz and the tuning process stopped when the
frequency was 19.98 kHz (well within the specified limits). The relationship
between frequency and diameter is fairly linear, as shown in figure 6.05.
Typically the frequency rises by about 90 Hz per 1 mm reduction in diameter.

Note that during the tuning process both the R1 and R3 frequencies also rise.
This is the reason why, in general, the problems experienced with
interference from the unwanted harmonic frequencies cannot be solved by a
change in the working frequency.

Note also that the circle diameter increased slightly as the tuning progressed.
There are two likely reasons for this. Firstly as metal is removed from the die
there is less moving mass and hence less energy loss. Secondly when the
resonant frequency of the system is low there is a significant mismatch
between the frequencies of the die and the transducer. Since the transducer
is forced to operate away from its resonant frequency the interface between
the die and transducer is not at an antinode (a stress node). This gives rise to
power losses from friction at the interface.

Items 10 to 20 are the measurements made while tuning a similar die for
necking a 45 mm aerosol can to 31 mm diameter. The die is again
constructed from a titanium alloy outer with a ferro-titanit insert. The tuning is
similar and the comments above apply equally well to this die also. The final
die diameter achieved is 2 mm greater than the earlier die, probably because
the inside diameter is smaller in this case.
## Table 6.03 - Admittance Plotter Measurements - Die Tuning (1 of 3)

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<td>162.3</td>
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**Notes:**
- Measurements - dies & mountings
- Dimensions: 595.0x841.0
In table 6.03 items 1 to 6 are the measurements made while tuning another die for necking 45 mm aerosol cans to 33 mm diameter. In this case a ferro-titanit insert has been fitted in an aluminium "diemounting" (combined die and tubular mounting in one piece). Note that the tuned diameter (166.5 mm) is considerably larger than the earlier die (152 mm). This was predicted by the finite element analysis (and could also have been predicted by the design graphs of chapter 4 and appendix 5), so the initial, as manufactured, diameter of the die was increased accordingly.

The power loss for this die (as indicated by the circle diameter) is comparable, or slightly better than, the earlier dies. Aluminium is inherently a less "lossy" material but extra losses were introduced by the mounting tube and these effects approximately cancel. This is a good result for the mounting, indicating a relatively small power loss. Mounting performance is discussed further in section 6.4 and results are given in tables 6.12 and 6.13.

Note how close the unwanted harmonic frequency appears in this die (20.964 kHz after tuning). This is the third harmonic (R3) frequency which for the two earlier dies appeared at about 21.7 kHz. This is a particular problem for this combination of materials and geometry, leading to "mode switching" in use (see section 3.3.2). In this case the problem can be avoided by simply using other combinations of materials as shown by the results for the other die in table 6.03 and both dies in table 6.02.

Items 11 to 16 in table 6.03 show the results for another die, in this case constructed from a titanium alloy diemounting with a Syalon insert (see section 3.7 for a detailed description of the materials used for these dies). This again shows a relatively large circle diameter (provided the die was held at the mounting flange not placed on its face) and the R3 frequency at 21.9 kHz is separated from the working frequency far enough to avoid mode switching. Two new circles have also appeared at 20.7 and 21.6 kHz but these are very small (1.5 and 1 mV respectively) and so would not be expected to cause any problems. These are probably resonances of the
mounting tube, remnants of the multiple resonant frequencies of the free mounting. Some tiny circles were often found in the circle plots of dies with mountings but circles of diameter less than 1 mV were generally not recorded.

Table 6.04 shows the results recorded while tuning two "shaped dies" (for further details see section 3.6). Items 1 to 9 are for a die made from aluminium alloy with a ferro-titanit insert with a profile suitable for necking 45 mm aerosol cans to 31 mm diameter. This is very similar to the die mounting (items 1 to 8 of table 6.03) which showed problems with the closeness of the R3 frequency. In fact this was deliberate. Knowing the problems with this combination of materials and geometry this die was constructed specifically for the purpose of testing a theoretical solution - the three-flat shaped die.

The initial results were recorded for the die before the three-flat shape was machined onto it. This shows the R0 and R3 frequencies where they would be expected (the R0 lower than 20 kHz before tuning, the R3 uncomfortably close). Using the rate of increase of the resonant frequencies observed for the earlier die during tuning an estimate of the tuned frequencies can be calculated. This calculation yields the following:

At diameter 170.3, R0 frequency 20.0 kHz, R3 frequency 20.76 kHz.

This would not be satisfactory because of the low R3 frequency, but in fact this tuning operation was not done. Instead the die was remachined to the three-flat shape. The three flats were made to equal dimensions, 74.0 mm from the centre of the die. This dimension was calculated to make the included angle of each flat 60° when the die diameter reached the predicted 171.0 mm (see figure 6.06). This procedure was used so that the tuning operation could be carried out by simple turning of the outside diameter of the die in a lathe, rather than repeated machining of the flats in a milling machine.
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<th>TUNING STATUS</th>
<th>VIBRATION MEASUREMENTS (FREQUENCIES /Hz, BANDWIDTH /Hz, CIRCLE DIA /mm)</th>
<th>COMMENTS / OTHER MEASUREMENTS</th>
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**TABLE 6.04 - ADMITTANCE PLOTTER MEASUREMENTS - DIE TUNING (3 OF 3)**
When the die was retested the change in measured performance was dramatic, as shown by the results of item 3. The separation between the R0 and R3 frequencies has been increased from 0.8 to 2.5 kHz, and at the same time the size of the R0 circle has increased while the R3 circle has shrunk. The prime aim of using the shaped die was to increase the frequency separation but both of these effects act to reduce the chances of mode-switching.

Remember, however that there is a price to be paid for this improvement in the frequency performance of the die. The axisymmetric (R0) mode becomes distorted as if by addition of some R3 mode, causing a variation in the vibration amplitude around the die. This is described in more detail in section 3.6.

The subsequent tuning of the die was accomplished as normal but some differences from the tuning of round dies were noted. The rate of increase of the R0 frequency as the diameter was reduced is less than normal (see figure

235
6.05). This would be expected because when machining down a shaped die less material is removed than from a round die (the flatted areas are not touched). Also the R3 frequency did not change significantly during the tuning process (ie the rate of increase of the R3 frequency is effectively zero). Finally note that the tuned outside diameter is greater than that predicted earlier for the round die. Extra material is needed between the flats to make up for what is lost.

The die described above was made as a test of the shaped die concept. For that application other material combinations were known which would give perfectly satisfactory results using the simple round die. There are other applications, however, for which no combination of the known materials would be satisfactory without using a shaped die. This is described in section 4.5.3. One such application is the necking of 66 mm beverage cans to 58 mm diameter. For this purpose a diemounting (combined die and mounting) was
constructed using an aluminium outer die with a syalon insert. This was the first real application of the three-flat concept.

The results for this diemounting are listed in table 6.04, items 12 to 18. Again measurements were taken before and after machining the three flats, and during subsequent tuning by reducing the outside diameter. Again the flats were specified with the aim of achieving a 60° included angle after tuning. The distance from the centre to each flat is 73 mm, giving a theoretical 60° included angle at 168.6 mm diameter. The tuned diameter of 169 mm gives an included angle of 60.5°. The listed results for this die are almost identical to those of the experimental three-flat die described above. The final measured performance (based on circle diameter and frequency separation) is perfectly satisfactory.

6.2.2.3 Admittance plotter measurements - dies and mountings

Table 6.05 shows miscellaneous measurements made on a wide variety of ultrasonic dies from all stages of the project. In this section their characteristics and performance will be compared.

Items 1 and 2 are measurements of a solid aluminium test die, taken first with the die placed on the bench on edge, and second with the die suspended by a piece of string through its centre. Comparing these measurements shows no significant change in the resonant and antiresonant frequencies but a significantly smaller circle for the die placed on edge and a correspondingly larger bandwidth. These two cases tend to represent the extremes of fixing losses - for dies placed on one face (the usual method of testing) a circle diameter somewhere in between would be expected.
<table>
<thead>
<tr>
<th>ULTRASONIC EQUIPMENT TESTED</th>
<th>DATE</th>
<th>DIE DIA (mm)</th>
<th>TUNING STATUS (mm)</th>
<th>VIBRATION MEASUREMENTS (FREQUENCIES /Hz, BANDWIDTH /Hz, CIRCLE DIA /mV)</th>
<th>COMMENTS / OTHER MEASUREMENTS</th>
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</thead>
<tbody>
<tr>
<td>01 Kerry 757 + Al Test die 1 (on edge)</td>
<td>9/86</td>
<td>155</td>
<td>complete</td>
<td>20091 9 19 20193</td>
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<td>complete</td>
<td>20087 6 29 20194</td>
<td></td>
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<tr>
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<td>complete</td>
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<td></td>
</tr>
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<td>155</td>
<td>complete</td>
<td>20019 6 6.5 20055</td>
<td></td>
</tr>
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<td>155</td>
<td>complete</td>
<td>9.5</td>
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<tr>
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<td>19852 17 4.5 19888</td>
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<td>07 Kerry 757 + MS die 9 (suspended)</td>
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<tr>
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</tr>
<tr>
<td>14 Kerry 757 + 45-33 Al/Nk Diemounting + can + plunger</td>
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<td>complete</td>
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<td></td>
</tr>
<tr>
<td>16 Tessonics + 45-33 Al/Nk Diemounting + can + plunger</td>
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**TABLE 6.05 - ADMITTANCE PLOTTER MEASUREMENTS - MISCELLANEOUS DIES AND MOUNTINGS**
Items 2 and 3 can be used to compare the test die (number 1) with another (test die 2) which was manufactured from a billet of aluminium which had been forged to give a grain structure which was expected to be more favourable to the radial vibrations. The forging was done by taking a 5 inch diameter bar of aluminium alloy and forging it axially to half its original length, causing the diameter to grow to about 7 to 8 inches. The same grade of aluminium was used for both dies and they were geometrically identical. Comparing items 2 and 3 it is clear that there is no significant difference between the measurements made of these two dies. It is possible that the modified grain structure of the forged die could make it more fatigue-resistant but this could not be verified without testing to destruction at high amplitude (which was not done).

Items 4 and 5 again show a comparison of measurements made with the die on edge and suspended. The die in this case is one of the early design dies intended for necking a 45 mm aerosol can to 25 mm diameter (this process was never successful). The die was constructed using an M2 insert in a stainless steel (316) outer. Neither of these materials has been commonly used in more recent dies, the M2 because alternatives are available which are harder or more convenient to use (see section 3.6) and the 316 because it is far more lossy than the preferred titanium and aluminium alloys. This is shown by the admittance circles which are less than half the size of those of comparable dies made in the preferred materials. Again a smaller circle results from placing the die on edge while taking the measurements.

Items 6, 7 and 8 are the results for two more early dies used for necking the 45 mm aerosol to 33 mm diameter. For a quick, cheap trial the first die was made from solid mild steel (die 9) and a duplicate was subsequently made in EN41 tool steel (die 9.1). The small circle diameters (5 to 7 mV) indicate heavy losses, particularly for the mild steel die.

It is interesting to compare the circle diameters measured for these early dies with the circle diameter of the transducer system which was used at the time.
Neglecting losses in the piezo-electric transducer (reasonable considering its much larger circle size) it can be said that the losses in the magnetostrictive transducer system are approximately three times greater than in the worst die (based on the ratio of circle diameters). This would indicate that the early dies were in fact perfectly adequate in that system. The need for more efficient dies only becomes apparent when an efficient piezo-electric transducer is used.

Items 10 to 16 show some measurements of the aluminium / ferro-titanit die for necking 45 mm aerosols to 33 mm diameter. The poor performance of this die (ie the closeness of the unwanted R3 frequency) has been discussed previously. In this series of tests the die was installed in the forming rig using an aluminium tuned mounting. It was not possible to measure admittance while forming a can (the necking process is too fast) but to simulate the type of loading this would impose on the die a fully formed can was pushed into the die and held there by the internal forming tool (plunger). Pressurized air in the pneumatic cylinder which operated the plunger ensured that a constant force (approx 3 kN) was applied to clamp the can into the die. Two types of transducer (Telsonic and Kerry) were also tested. The aim of these measurements was to gain a better understanding of the problem of mode switching (particularly for this die) and why the Telsonic system did not seem to suffer from it.

Without the can and plunger loading the die the results are mostly similar for the two transducers. In each case the R0 circles (at approx 20.0 kHz) are satisfactory, and two unwanted frequencies appear, close together at approx 21.0 kHz. Another harmonic frequency appears at 24.0 kHz in both cases. The interesting feature here is the pair of harmonic frequencies at 21 kHz. Analysis predicts only the R3 near this frequency, but in general a pair of resonant frequencies could be expected for every non-axisymmetric resonance. One of the resonant frequencies would correspond to the R3 mode aligned with an antinode at the transducer, and the other aligned with a node at the transducer.
Normally the first mode should be driven by the transducer and the second one filtered out because the die motion will not match that of the transducer. This effect is also discussed in section 2.4.1. The only significant difference between the results for the two transducers concerns the relative sizes of the circles for the pair of R3 frequencies. For the Telsonic transducer the lower frequency resonance has a very small circle diameter (2 mV) but using the Kerry transducer both circles are relatively large (12 and 7 mV). This indicates that the transducer is vibrating unevenly, at least for some of the time, and this may have contributed to the mode-switching problem. Figure 6.07 shows (much exaggerated) how the two modes could look and the sort of uneven motion the transducer would need in order to excite them.

Comparing the results for the die with the can and plunger inside it the two transducers again give very similar results, again with the exception of the relative sizes of the two R3 frequencies. Using the Telsonic transducer one of
the R3 pair is large (15 mV) while the other is small (1 mV), but using the Kerry transducer the two circles are again of similar size (11 and 8 mV).

Comparing results for the die loaded (by the can and plunger) and unloaded the following observations can be made: The circle diameter for the R0 mode is reduced when the die is loaded.

The circle diameters for the R3 mode pair are not much reduced, particularly for the Kerry transducer.

The R0 resonant frequency is increased by about 40 Hz when the die is loaded.

The resonant frequencies of the R3 pair are not increased much (on average 10 Hz) when the die is loaded.

All of these effects will, in general, tend to promote mode switching by bringing the unwanted resonances closer to the working frequency and increasing the size of the unwanted resonance relative to the R0. The magnitude of the changes is small in proportion to the relatively small amount of loading applied by the plunger (compared to the necking process). Nevertheless trends are apparent which must tend to destabilize the R0 mode, and the Kerry ultrasonics system is measurably less stable than the Telsonic. Whether mode switching takes place will depend not only on the stability of the transducer and die (as indicated by these measurements) but also on the quality and stability of the control system used to maintain resonance. The fact that mode switching has been observed in this die using the Kerry system indicates that for this equipment the limit has been reached. For the Telsonic equipment (which probably also employs a superior control system) the limit has not yet been found.

Finally, items 18 and 20 of table 6.05 show measurements of the aluminium diemounting with sylcon insert for necking 66 mm beverage cans to 58 mm diameter. The tuning of this die was described in the previous section (items 12 to 18 of table 6.04). These measurements differ only in that the
diemounting was clamped by its flange. Comparing these measurements with the final tuning measurement shows no significant differences, indicating that clamping the flange has minimal effect on the die vibrations. Comparing the results for the two transducers shows no major differences.

6.2.2.4 Admittance plotter measurements - mounting evaluation

Tables 6.12 and 6.13 show evaluations of the performance of a number of tubular tuned mountings. These are discussed in section 6.4.

6.2.3 Comparison with FE predictions

For the results shown in tables 6.02 to 6.04 (the die tuning operations) corresponding information from the finite element analysis has been summarized in tables 6.06 to 6.11 to give an indication of the normal levels of accuracy. In each case the type of finite element model and the number of elements and master degrees of freedom is indicated, followed by the predicted natural frequencies (for the die alone) categorized by the system of mode nomenclature described in section 3.1. The actual outside diameter and measured frequencies (as also shown in tables 6.02 to 6.04) have also been included in heavy type. Note that for some models (the round versions of the three-flat dies) no real measurements have been included because no equivalent die was produced.

Tables 6.06 and 6.07 show the analysis of conventional (round) dies. This type of analysis is straightforward and efficient using two-dimensional axi-symmetric harmonic elements (see section 2.4.3 for a full description of this type of element). The number of elements ranges from 52 to 62 and the number of masters from 100 to 130. Comparing the predicted and actual results shows that the predictions were generally good, with the maximum difference on diameter only 1.3 mm and the agreement on R0 and R1.
frequencies well within 1%. The only significant error is the consistent overestimating of the R3 frequency (by about 0.5 to 1 kHz, or 2.5 to 5%). There are two sources of error contributing to this. Firstly the tendency of the FE model to overestimate the frequencies, particularly of the harmonic modes (as discussed in detail in section 2.8). Secondly the finite element model does not include the transducer, whereas all measurements inevitably do. The effect of the transducer is to "pull" frequencies towards its own working frequency, 20 kHz. These two sources of error will tend to cancel out for frequencies below 20 kHz (typically the R1 frequencies) but will add together for frequencies above 20 kHz (ie the R3).
### Finite Element Model

<table>
<thead>
<tr>
<th>MODEL DETAILS</th>
<th>PREDICTED MODES AND FREQUENCIES</th>
<th>COMMENTS</th>
</tr>
</thead>
<tbody>
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<td></td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>Titanium / Nikro die Ø45 - Ø33</td>
<td>151.0</td>
<td>152.0</td>
</tr>
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<td>ELEMENT NAME</td>
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<td>NO OF MASTERS</td>
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<td></td>
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</tbody>
</table>

| Aluminum / Nikro diemounting Ø45 - Ø33 | 167.0 | 166.5 | MODEL OF: CROSS-SECTION (2D) | 0 | 9616 | 20033 | 19971 | Unidentified small resonance also found at 21116 kHz |
| ELEMENT NAME | STIF83 | ELEMENT TYPE | 8-NODED QUADRILATERAL | 1 | 17049 | 16684 | | |
| NO OF ELEMENTS | 56 | | | 2 | 6099 | 11893 | 24209 | 26793 | |
| NO OF MASTERS | 130 | | | 3 | 12814 | 21415 | 20964 | | |
| | | | | 4 | 19479 | 28128 | | | |
| | | | | 5 | | | | | |

**Table 6.06 - Comparison of FE predictions with admittance plotter measurements**
<table>
<thead>
<tr>
<th>FINITE ELEMENT MODEL</th>
<th>DIE DIA (mm)</th>
<th>MODEL DETAILS</th>
<th>PREDICTED MODES AND FREQUENCIES</th>
<th>COMMENTS</th>
</tr>
</thead>
<tbody>
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<td></td>
<td>MODEL OF...</td>
<td>T TORSIONAL</td>
<td>R RADIAL</td>
</tr>
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<td>18462</td>
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<td></td>
<td>NO OF ELEMENTS</td>
<td>4 21729</td>
<td>21729</td>
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<td></td>
<td></td>
<td>NO OF MASTERS</td>
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<td>Titanium / Syalon die mounting #2 (\phi 45 - \phi 31)</td>
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<td></td>
<td>NO OF MASTERS</td>
<td>5 20902</td>
<td>29960</td>
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</table>

Unidentified small resonances also found at 20713 kHz and at 21645 kHz.
Tables 6.08 to 6.11 show the results for the FE models of the two 3-flat designs (for which the measured results are shown in table 6.04). Modelling this type of die is more difficult because the axi-symmetric elements are no longer applicable. A similar analysis of the die using a full three-dimensional model was required but limitations on the computing power available made this impossible to analyse. As a compromise a simpler 3-D model with a coarse mesh was analysed and the results compared with 2-D axi-symmetric models.

The results of this analysis for the experimental Aluminium / Nikro die are shown in tables 6.08 and 6.09. Table 6.08 (top) shows the axi-symmetric model similar to those used for the four round dies described previously. In this case the predicted R3 frequency was 21.2 kHz (lower than any previous die). This is well within the "danger zone" around 20 kHz and it is likely that the actual frequency would have been even lower, for the reasons described above. This was confirmation that the 3-flat design was required.

Table 6.08 (bottom) shows the results for the same model and the same axi-symmetric elements but with a much coarser mesh (only 18 elements compared to the 62 used earlier). This was done because it was known that a coarser model would be needed for the 3-D analysis and the coarse mesh 2-D model would provide a better comparison. Comparing the results for the two 2-D models shows that all the coarse model frequencies were higher (less accurate), and the largest discrepancies were in the torsional mode frequencies and the higher harmonic radial modes. The predicted R3 frequency was increased by 0.5 kHz in the coarse model, indicating the necessity of a reasonably fine mesh.
### Table 6.08 - Comparison of FE Predictions with Admittance Plotter Measurements

<table>
<thead>
<tr>
<th>FE Model Details</th>
<th>predicted Modes and Frequencies</th>
<th>Comments</th>
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</thead>
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<td><strong>Die Dia (mm)</strong></td>
<td><strong>Cross-section (2D)</strong></td>
<td><strong>Element Name</strong></td>
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</table>

*This rough model is approximately the later 3D models.*
Table 6.09 (top) shows the results for the 3-D model round die. This was equivalent to the two axi-symmetric models and permitted a further comparison of probable accuracy of the different models. The predicted frequencies for the 3-D model were all higher again (even less accurate) than those of the coarse 2-D model and in this case the predicted R3 frequency was 22 kHz which, if true, would have been acceptable.

Table 6.09 (bottom) shows the results for the 3-D model 3-flat die. Knowing the inaccuracy of these models, these results were treated with caution, but comparison with the round die model was expected to indicate the true results. Comparing the most important (for this die) RO and R3 frequencies, the effect of the 3-flat design is to raise the RO slightly (by 0.3 kHz) and to raise the R3 considerably (by 1.8 kHz). This indicated that the separation between RO and R3 frequencies should be increased by 1.5 kHz.

The measured results for this die are also shown in table 6.09 (bottom). Comparing these, the predicted die diameter is 2.5 mm less than the actual (not enough allowance was made for the flats increasing the RO frequency) and the R3 frequency was, as expected, greatly overestimated (by 1.6 kHz). The most important point to note, however, is that the R3 frequency, at 22.3 kHz, was fully acceptable. The 3-flat design had increased the frequency separation as predicted.
<table>
<thead>
<tr>
<th>FINITE ELEMENT MODEL</th>
<th>DIE DIA (mm)</th>
<th>MODEL DETAILS</th>
<th>PREDICTED MODES AND FREQUENCIES</th>
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<th>FINITE ELEMENT MODEL</th>
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TABLE 6.09 - COMPARISON OF FE PREDICTIONS WITH ADMITTANCE PLOTTER MEASUREMENTS
A similar procedure was used for the design of a die for necking 66 mm beverage cans to 58 mm diameter. As discussed in section 3.7, choosing alternative materials for the construction of this die did not produce any results which would be acceptable. The use of a shaped die was therefore essential, and the predicted natural frequencies indicated that the (3-flat) design would be suitable. The results in tables 6.10 and 6.11 are for three models, one 2-D axi-symmetric (coarse mesh), one 3-D round and one 3-D with flats. As before the frequencies are significantly overestimated by the 3-D models but increased separation between R0 and R3 frequencies is achieved, much as predicted. During tuning the die diameter was machined to exactly the size predicted (169 mm) and the measured R3 frequency was then found to be 22.2 kHz - fully acceptable.
### Table 6.10 - Comparison of FE Predictions with Admittance Plotter Measurements

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<td><strong>Model Details</strong></td>
</tr>
<tr>
<td>Aluminium / Syalon die &amp; mounting Ø65 - Ø58</td>
<td>169.0</td>
<td><strong>Model of...</strong></td>
</tr>
<tr>
<td></td>
<td></td>
<td><strong>Element Name</strong></td>
</tr>
<tr>
<td></td>
<td></td>
<td><strong>Element Type</strong></td>
</tr>
<tr>
<td></td>
<td></td>
<td><strong>No of Elements</strong></td>
</tr>
<tr>
<td></td>
<td></td>
<td><strong>No of Masters</strong></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Finite Element Model</strong></td>
<td><strong>Die Dia.</strong> (mm)</td>
<td><strong>Model Details</strong></td>
</tr>
<tr>
<td>Aluminium / Syalon die &amp; mounting Ø65 - Ø58</td>
<td>169.0</td>
<td><strong>Model of...</strong></td>
</tr>
<tr>
<td></td>
<td></td>
<td><strong>Element Name</strong></td>
</tr>
<tr>
<td></td>
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<td><strong>Element Type</strong></td>
</tr>
<tr>
<td></td>
<td></td>
<td><strong>No of Elements</strong></td>
</tr>
<tr>
<td></td>
<td></td>
<td><strong>No of Masters</strong></td>
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</table>
### Table 6.11 - Comparison of FE Predictions with Admittance Plotter Measurements

<table>
<thead>
<tr>
<th>Finite Element Model</th>
<th>Die Dia (mm)</th>
<th>Model Details</th>
<th>Predicted Modes and Frequencies</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aluminium / Sylon dies mounting Ø65 - Ø58 With 3 x 60° Flats</td>
<td>169.0 169.0</td>
<td>Model of....</td>
<td>0 Torsional 20484 19986</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Element Name</td>
<td>1 19600 18273</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Element Type</td>
<td>2 6693 10863 27324</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>No. of Elements</td>
<td>160</td>
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</tr>
<tr>
<td></td>
<td></td>
<td>No. of Masters</td>
<td>23176 33104</td>
<td></td>
</tr>
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</table>

**Finite Element Model**

![Finite Element Model Diagram](image)

**Model Details**

- **Model of...**: HALF DIE (3D)
- **Element Name**: STIF45
- **Element Type**: 8-NODED BRICK
- **No. of Elements**: 160
- **No. of Masters**: 400

**Predicted Modes and Frequencies**

- **Torsional**: 12045 20484 19986
- **Radial**: 20484 19986
- **Rotational**: 18273
- **Axial**: 27324

**Comments**

-None-
6.2.4 Accuracy of FE predictions

The potential accuracy of the FE models is discussed in section 2.8, but comparison with real measurements (as described here) introduces some further sources of error:

1) Tolerances on dimensions and variations in material properties may result in the die failing to match its finite element model perfectly.

2) The presence of a transducer on the die during frequency measurements will itself affect the resonant frequencies.

From the results described in the previous section it appears that these sources of error can account for up to about 3 mm discrepancy in the tuned outside diameter of a typical die. To allow for this the dies are initially manufactured about 6 mm oversize and then tuned by gradual machining down. The 6 mm "tuning allowance" could be reduced but there is little advantage in doing this because the dies are usually manufactured by turning down a stock size bar (eg 7" diameter).

The two-dimensional axi-symmetric harmonic elements offer good accuracy and efficiency.

To analyse the shaped dies only a full three-dimensional model is suitable, but limitations on available computing power required the use of a much coarser mesh for this type of model which was far less accurate. The technique adopted, using both 2-D and 3-D models gave satisfactory results.

Harmonic frequencies above 20 kHz tend to be overestimated by the FE model, while harmonic frequencies below 20 kHz are generally reasonably accurate.
6.3 **ESPI MEASUREMENTS**

One of the most powerful techniques used to evaluate the performance of ultrasonic dies was Electronic Speckle Pattern Interferometry (ESPI). The technique was developed for use on ultrasonic dies by Tyrer and Shellabear [117], [118], [119], [120] along with others at Loughborough University during the course of the SERC-sponsored project. This non-contact measuring system was capable of giving quantitative data for the vibrations of the whole die in real time.

There are however some disadvantages of this system compared to the other systems described previously, particularly the cost and setting up time. Another potential problem is the interpretation of the results produced by ESPI which are not always immediately understandable. To assist in identifying modes of vibration it was necessary to process the finite element results to produce an image equivalent to an ESPI picture.

This section describes the general principles of ESPI and the techniques used to interpret the results by comparison with finite element models.

### 6.3.1 Principles of ESPI

For a full explanation of the technique see the various publications of Shellabear, Tyrer et al [117], [118], [119], [120]. The die is illuminated by an interfering pair of laser beams, producing a speckle pattern. A video camera is used to digitally record an image of this pattern. Any movement of the die surface causes changes in the speckles, depending on the distance moved. By digitally subtracting the new image from the previous stored one, the changes are shown, with light and dark fringes showing areas where the amount of movement is similar (like contour mapping). By different arrangements of the laser beams the equipment can be made sensitive to movements along three orthogonal axes (ie with the die set up on a horizontal axis, movements can be measured out-of-plane, in-plane horizontal and in-plane vertical).
Figure 6.08 shows a typical arrangement.

For identifying pure modes of vibration the out-of-plane image is useful because in most cases the axial motion gives a good idea of the nature of the mode (i.e., its harmonic number and whether it is predominantly radial or torsional). However, it does not show directly the most useful component of vibration (the radial component), and where there is a mixture of two modes one (with predominantly radial motion) may be masked by the other (with significant axial motion).

The two in-plane images will provide the extra information required but they are much more difficult to interpret because they show elements of both radial and hoop motion in different areas of the die. If amplitude and phase information is known then the precise motion of the die at any point can be evaluated from the three images (and Shellabear [120] has done this) but the process is laborious and often the full set of results is not available. The alternative chosen for much of this work was to process the finite element
results for each mode into simulated ESPI images which could be compared with the real ESPI images as they were generated.

When this was done it was found that some images matched well but others seemed to contain elements of the FE images for more than one mode of vibration. It is to be expected that the modes of vibration of a real die, in the presence of imperfections and damping, will not be pure but in general will be combination modes made up of a number of pure modes.

To allow for this in the interpretation of the ESPI results a program was produced to process the FE results, not only converting them to ESPI type images but also allowing the combination of a number of different (pure) modes in proportions chosen by the user. Thus using the program and selecting mode combinations by trial-and-error the user could find a combination which produced a set of images to match the real ESPI results. While not positive proof of the nature of the vibrations of the real die this gives a very good indication.

6.3.2 Converting FE results to simulated ESPI plots

6.3.2.1 Pure modes

The Ansys program produces a huge amount of data for even the simplest model. To avoid wasting processing time and storage the relevant finite element data was first extracted and stored in a series of data files. Each file included a set of data (the radius and three components of displacement) for all nodes along a radial line on one face. The data for each mode was stored in a separate file.

Note that a 2-D axisymmetric-harmonic finite element model was used (section 2.4.3). A 3-D model could have been used but the 2-D model is more efficient. The variation in the components of amplitude over the surface of the die can be accurately predicted using the known harmonic variation as
described in section 2.4.3 (the harmonic number is also stored on the data file).

The main program was written in Fortran using the Issco Codebook prototype [123]. This is a program which takes the user through a question and answer session and then automatically generates the Fortran source code to produce the required plots by calling subroutines in the "Disspla" library. The technique is similar to that of example 3.6.6 (appendix 3) for making contour plots of the performance of different die designs. After the plotting program is generated it can be edited to perform other functions (in this case data processing) and recompiled like any other program. The final result is a program which is less tidy than one written specifically for the application but which takes less effort to produce.

To display a pure mode the program must read in the data for that mode from the appropriate data file and then use the data to calculate the component of amplitude in a chosen direction (axial, horizontal or vertical) at points all over the die face. The result is stored in a 2 x 2 array and subsequently displayed using colours corresponding to the light and dark fringes produced by ESPI.

The calculations must take into account:

1) The radial and angular position of the point.
2) The harmonic number.
3) For horizontal and vertical components: both radial and hoop amplitudes.
4) For harmonic number greater than 0: The angular alignment of hoop components of amplitude (hoop amplitude is zero at the radial / axial antinode).

6.3.2.2 Distorted modes

For distorted modes (i.e., combinations of two or more pure modes) the procedure is the same but the components of amplitude for each mode must be added together.
The program in its latest form allowed the user to select one main mode and one secondary mode. The reference angle for each one can be specified (i.e., where on the image the mode's antinodes should appear) and for the secondary mode the fraction of that mode which should be added. Future enhancements may include the option to add a third mode if this becomes necessary.

6.3.3 Comparison of results

Figures 6.09 to 6.13 show typical ESPI-equivalent images generated from the finite element data and actual ESPI photos from Shellabear's work. The computer-generated images correspond to the pure modes which are generally of interest, because they typically appear at a frequency close to 20 kHz. The photos (where available) show the closest measured response modes of the die.

Figure 6.09 ESPI images (predicted and actual) - R0 mode
(ESPI photos not available for this mode)

Figure 6.10 ESPI images (predicted) - R1 mode

Figure 6.11 ESPI images (predicted and actual) - R3 mode
Figure 6.12 ESPI images (predicted) - T4 mode

Figure 6.13 ESPI images (predicted and actual) - D2 mode
These results have been produced for an aerosol necking die comprising a ferro-titanit insert in an aluminium outer (die materials are discussed in section 3.6). Other die designs produce ESPI-equivalent images which are generally similar.

Photos taken from the ESPI video pictures are shown in references 1 (for R0, R3 and D2 modes) and reproduced here, where available, along with the corresponding simulated ESPI image. This allows easy comparison of the theoretical vibrations with the real measurements.

For the R0 and D2 modes (figures 6.09 and 6.13) the correlation with the pure mode FE results is clear, although some vertical distortion of the fringes is apparent. This is to be expected particularly for the D2 mode because the frequency (24 kHz) is significantly separated from the 20 kHz working frequency. This means that the transducer unit which drives the die must be detuned and therefore effectively applies a mass to the outside of the die moving in a vertical direction. Nevertheless the images match well enough to clearly identify both modes.

For the (presumed) R3 mode (figure 6.11) the correlation is less clear. It was believed to be a distorted R3 mode because its natural frequency (20.8 kHz) approximately matched the FE prediction and because the image for axial motion shows a three-way symmetry (although for the pure R3 mode a six-way symmetrical image would have been expected). The closest other mode to this one is the R0 at 20 kHz so it was expected that the distortion would take this form. Figures 6.14 to 6.16 show different combinations of R3 and R0 modes. In figure 6.14 the R0 mode at half-amplitude has been added, but this shows a distortion opposite to that shown by the real measurements. Figure 6.15 shows the R0 mode at half amplitude subtracted from the R3, and this shows good agreement with the measurements. Note that for each mode the amplitude is normalized but the sign is purely arbitrary. Figure 6.16, shows the combination R3 - 0.2 x R0. Judging by eye, the best correlation is probably somewhere between the last two, i.e. R3 - 20 to 50% R0.
Figure 6.14 ESPI images - Combination R3 + 0.5 R0 mode

Figure 6.15 ESPI images - Combination R3 - 0.5 R0 mode
6.3.4 ESPI Summary

A method of processing finite element results has been demonstrated which produces images equivalent to those of the real die produced using electronic speckle pattern interferometry (ESPI).

When used for comparison with the images of a vibrating die these simulated ESPI images allow interpretation of the mode of vibration.

For cases where the mode of vibration is not a pure mode its composition may be evaluated by comparison with an image produced for two pure modes combined in a specified ratio.
6.4 EVALUATION OF TUBULAR MOUNTING PERFORMANCE

Between the initial concept for the thin walled tubular mounting (described in chapter 5) and the final optimized design, several attempts were made to measure its performance. These will be described in turn.

6.4.1 Free vibrations of steel prototype

The first prototype was manufactured in mild steel (for minimum cost). This was a tube of inside diameter 80 mm, outside diameter 85 mm and length 130 mm with a central flange as shown in figure 6.17. This corresponds to the initial concept of a tube resonant simultaneously in both radial and longitudinal modes. Finite element analysis predicted a large number of free vibration modes in the region of 20 kHz. An experiment was planned to measure these frequencies.
In order to excite vibrations in the die while it was not connected to the die a small piezo-ceramic disk was fixed (using epoxy adhesive) to the free end of the mounting. This was driven electrically by a small variable frequency generator. Motion of the tube walls was sensed by a noise level meter mounted at a fixed distance from the mounting. Thus sensing was entirely non-contact but driving the vibrations involved the addition of a small mass to the system which would inevitably affect the frequencies. The effect was minimal, however, because of the small volume and relatively low density of the piezo-ceramic disk compared to the steel tube.

The measured results are shown in figure 6.18. The points on the resonance curve have mostly been confined to resonances (peaks) and antiresonances (troughs) because of the huge number of resonances involved. Identification of the modes is not possible because of the method of measurement, but the nature of the resonance curve was generally as predicted (i.e., many resonances around 20 kHz).
Indeed the number of measured resonances would have been doubled relative to the axi-harmonic FE model because the piezo-ceramic disk would slightly "unbalance" the tube creating a pair of resonances for each of the theoretical ones.

Further trials on the free mounting were not expected to yield any more useful results so all subsequent testing was done on the combined system (die and mounting).

6.4.2 Admittance plotter measurements of mounting performance

The admittance plotter (described in section 6.1.2) gives an accurate and sensitive measure of the characteristics of a mechanical system comprising a transducer and other components connected to it. In order to evaluate the effect of the mounting it was necessary to compare the transducer, die and mounting against the transducer and die alone. The results of this and some other comparisons for several dies and mountings are recorded in tables 6.12 and 6.13, and will now be discussed in turn.

All the measurements listed in table 6.12 are for the aluminium test dies with various early mountings fitted. Other measurements on these dies (section 6.2.2.3 and table 6.05) showed that the dies were effectively identical and that circle diameters for either die with a Kerry transducer would range from 19 to 30 mV depending on the position of the die (resting on the bench or hanging on a string etc).

Items 1 to 4 show some measurements of the test die fitted with the first aluminium mounting. This was dimensionally identical to the steel mounting (ie not optimized) and fitted to the back of the die by screws through a flange (figure 6.19). The measured circle diameters are significantly less than for the die alone, and the smallest circle (10 mV) was measured with the mounting flange clamped in place. This indicates that power losses from this system would be high.
<table>
<thead>
<tr>
<th>ULTRASONIC EQUIPMENT TESTED</th>
<th>DATE</th>
<th>DIE DIA (mm)</th>
<th>TUNING STATUS (mm)</th>
<th>VIBRATION MEASUREMENTS (FREQUENCIES /Hz. BANDWIDTH /Hz. CIRCLE DIA /mm)</th>
<th>COMMENTS / OTHER MEASUREMENTS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kerry 757 + Al Test die 2 (suspended)</td>
<td>9/86</td>
<td>155</td>
<td>complete</td>
<td>20053 6 30 20166</td>
<td>Aluminium mounting 1 was not optimized ie. Ø80, Ø65, 130 long with central flange. Fixing to die was by flange + 8 screws</td>
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<tr>
<td>Above + Al mtg 1 (resting on face)</td>
<td></td>
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<td>19887 8 14 19951</td>
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<td>19887 8 14 19951</td>
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<td>Above + Al mtg 1 (hand held at flange)</td>
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<td></td>
<td>complete</td>
<td>19887 8 14 19951</td>
<td></td>
</tr>
<tr>
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<td>155</td>
<td>complete</td>
<td>20091 9 19 20193</td>
<td>Steel mounting was not optimized ie dimensions as above</td>
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<tr>
<td>Above + steel mtg (fixed by 8 screws)</td>
<td></td>
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<td>19674 3 5 19729</td>
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<tr>
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<tr>
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<tr>
<td>Above + steel flange ring (no screws)</td>
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<td>19945 13 19729</td>
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</tr>
<tr>
<td>Above + Al flange ring 1 (1 screw)</td>
<td></td>
<td></td>
<td>complete</td>
<td>19945 13 19729</td>
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<tr>
<td>Above + Al flange ring 1 (2 screws)</td>
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<td>19945 13 19729</td>
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</tr>
<tr>
<td>Above + Al flange ring 1 (4 screws)</td>
<td></td>
<td></td>
<td>complete</td>
<td>19945 13 19729</td>
<td></td>
</tr>
<tr>
<td>Above + Al flange ring 1 (8 screws)</td>
<td></td>
<td></td>
<td>complete</td>
<td>19945 13 19729</td>
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</tr>
<tr>
<td>Above + Al flange ring 2 (4 screws)</td>
<td></td>
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<td>Flange ring 2 is clearance fit in die</td>
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<tr>
<td>Above + Al flange ring 2 (2 screws)</td>
<td></td>
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<td>complete</td>
<td>20033 13 9 20223</td>
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<td></td>
<td></td>
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<td>20033 13 9 20223</td>
<td></td>
</tr>
</tbody>
</table>

| TABLE 6.12 - ADMITTANCE PLOTTER MEASUREMENTS - MOUNTING EVALUATION (1 OF 2) |
Items 5 to 10 show measurements of the other test die with the first steel mounting fitted. These show two problems. Firstly there were three measurable resonances around 20 kHz - this means that the mass of the mounting is great enough for its own resonances to seriously influence the whole system (particularly because the die is aluminium and hence relatively light). Secondly the circle diameters are all very small (at best only half of the smallest circle for the aluminium mounting). The three measurements were made to estimate the effect of the fixing screws. There was no significant difference when using eight, three or no screws.

Items 12 to 21 take these experiments further with three rings (one steel and two aluminium) which fitted into the back of the die in the same way as the mounting. All measurements were made with the die on its edge for direct comparison with item 5. When the steel ring was fitted (without screws but using an interference fit) there were significant additional losses (circle diameter reduced from 19 to 12.5 mV). Similar results were obtained for the first aluminium ring, which was also an interference fit in the die. In this case the fixing screws were also used and the circle plotted for one, two, four and all eight screws inserted. With the screws inserted both the resonant frequency and the circle diameter decreased indicating that the screws were adding both extra mass and extra losses. The second aluminium ring was a clearance fit in the back of the die. When this was fixed by four or eight screws its results were identical to those of the first aluminium ring, but when it was held by only two screws (fitted on opposite sides) the circle diameter became very small (1.5 mV) and held by only one screw the resonance was effectively eliminated.

This series of trials demonstrated that the mounting flange screwed to the back of the die was generally undesirable. The simple interference fit was better but even this was not ideal. For later dies and mountings the fixing system was redesigned as shown in figure 6.19. The flange and fixing screws were eliminated in favour of a simple interference fit. Furthermore the end of
the mounting tube was increased in thickness to increase its rigidity (and so improve the effectiveness of the interference fit) and at the same time maintain the 20 kHz resonance which had been lost when using the flange. After design optimization the interface was eliminated altogether by manufacturing the die outer and the mounting together in one piece.

Table 6.13 shows some further admittance plotter measurements used for evaluation of tubular mountings. Items 1 and 2 are results of circle plots for a die with and without an aluminium mounting fixed by interference fit only. The results show that the effect of the mounting is minimal. Items 4 and 7 (for the same die and mounting, but using a different transducer and a booster) show that the mounting flange can be fixed with minimal effect on the die performance.
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<th>DIE DIA (mm)</th>
<th>TUNING STATUS (mm)</th>
<th>FREQUENCY (Hz)</th>
<th>BANDWIDTH (Hz)</th>
<th>CIRCLE DIA (mm)</th>
<th>ANTIRIES FREQ (°)</th>
<th>COMMENTS / OTHER MEASUREMENTS</th>
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<tr>
<td>Kerry 757 + 45-33 Al/ M2</td>
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<td></td>
<td></td>
<td>19947</td>
<td>14</td>
<td>17</td>
<td>20072</td>
<td>21033</td>
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<tr>
<td>Above + Al mtg #2 (free)</td>
<td></td>
<td></td>
<td></td>
<td>19945</td>
<td>12</td>
<td>17</td>
<td>20075</td>
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<td></td>
<td></td>
<td></td>
<td>20008</td>
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<td>29</td>
<td>20075</td>
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</tr>
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<td>+Al mtg #2 (free)</td>
<td></td>
<td></td>
<td></td>
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<td>28</td>
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<td>1</td>
</tr>
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<td></td>
<td></td>
<td>20860</td>
<td>4</td>
<td>4</td>
<td>20910</td>
<td>4</td>
</tr>
</tbody>
</table>

TABLE 6.13 - ADMITTANCE PLOTTER MEASUREMENTS - MOUNTING EVALUATION (2 OF 2)
Subsequent mountings were made integral with the die, to eliminate potential losses at the interface. Some of these were described in sections 6.2.2.2 (die tuning) and 6.2.2.3 (miscellaneous dies and mountings). The good measured performance of these "diemountings" in comparison with the earlier dies is evidence for the success of the design.

6.4.3 Tubular mounting measurements - summary

These results show that in general the mounting has little effect on the resonant frequency of the system (presumably it is dominated by the greater momentum of the die) but adds some power losses to those of the remainder of the system. Additional losses for early designs running at 20 kHz, 10 μ amplitude were approx 300 W. Later versions under the same circumstances would use less power (of the order 100 W) but this was difficult to evaluate because in these later dies the mounting was made integral with the die outer (no shrink fit) so direct comparison with and without mounting was not practical.

This type of mounting (particularly in the later versions with optimized geometry) has therefore been shown to be a practical and efficient method of mounting the ultrasonic die.
6.5 MEASURED PROCESS FORCES

The process of necking welded cans (ie reducing the diameter of one end) is discussed in detail in section 1.1, along with the various problems which can occur. In this application the prime reason for using ultrasonics is to prevent collapse of the can body. Without ultrasonics the forming force exceeds the strength of the body, which is then crushed as shown in figure 6.20 (as figure 1.09). With ultrasonics the forming force is reduced (for reasons discussed in section 1.2) to less than the collapse force of the can body. The collapse of the can is therefore a crude measure of the process force.

Clearly the crushing / not crushing of the can was not an adequate method of force measurement and a more accurate method was required. In fact two methods of measuring force were used, as follows.

Figure 6.20 Cans necked successfully and crushed during necking
6.5.1 Evaluation of forming force in early work

In the early part of the project a very simple experiment was conducted. The hydraulic pressure supply to the cylinder used to perform the necking process was reduced so that minimal force would be applied to the can. One can was necked at this pressure and the necking process stopped when the required forming force exceeded the available force (hydraulic pressure x piston area). The neck diameter of the partially necked can was then measured. After a small increase in the supply pressure the process was repeated, and this continued until the available force was sufficient to crush the cans (full forming was not possible at this stage). The result was a set of corresponding forming force and neck diameter data which could be plotted as a graph.

ESTIMATED STRENGTH OF CAN = 7.1 kN

THEORETICAL RELATIONSHIP ASSUMING ZERO FRICTION (UPPER AND LOWER BOUNDS)

KEY

<table>
<thead>
<tr>
<th>LINE TYPE</th>
<th>DIE 2 ( \Theta 45-\Theta 25.4 )</th>
<th>DIE 3 ( \Theta 45-\Theta 30.7 )</th>
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<tr>
<td>WITHOUT ULTRASONICS</td>
<td>-</td>
<td>+</td>
</tr>
<tr>
<td>WITH ULTRASONICS</td>
<td>- - -</td>
<td>o</td>
</tr>
</tbody>
</table>

FIGURE 6.21 - MEASURED FORMING FORCE VS DIAMETER (EARLY WORK)
Figure 6.21 shows the results for cans formed with and without ultrasonics, in dies designed for a final diameter of 31 mm and 25.4 mm. For comparison a graph of the theoretical forming force is also included (calculated using the analysis shown in appendix 1). The analysis takes account of work hardening of the metal (based on measured strength before and after necking) but assumes zero friction. It is remarkable how closely the measured forming force for the cans formed with ultrasonics follows the theoretical curve for the initial part of the forming process (arguably indicating that the process happens with almost zero friction in the early stages). Note that towards the end of the process the measured forces (with ultrasonics) diverge from the theoretical curve but remain significantly lower than the measured forces without ultrasonics. Note also that a smaller diameter was achieved using the 25.4 mm diameter die, almost certainly because extra work is required to bend the material up into the straight section, and because contact with the (non-vibrating) inner tool causes extra friction. This straight section is necessary to provide the material for forming the rolled end ("curl").

6.5.2 Evaluation of forming force in later work

One problem with the method described above is that the can is formed slowly (coming to a complete halt). It has been found that the can is less likely to be crushed if the necking tools move at high speed (there is presumably a limit to this but this was not found within the speed capability of the hydraulic rams - about 0.25 m/s). It was therefore desirable to measure the forming force while the operation proceeded at normal speed. A load cell would have been ideal but this would have been difficult to fit into the forming rig because of various other moving parts. A simpler option was therefore implemented as shown in figure 6.22. Pressure transducers were fitted in the supplies to the hydraulic cylinder which powered the die movement. Two transducers were required to measure pressure above and below the piston.
Figure 6.22 - Equipment to measure forming forces on test rig.
Forming force was calculated by subtracting (pressure x area) below the piston from (pressure x area) above. An LVDT (linear variable displacement transducer) was also used to monitor the position of the moving die and so provide a reference to the progress of the forming process.

Results are shown in figure 6.23, for cans formed with and without ultrasonics. Note that the can formed without ultrasonics was crushed (causing a sudden drop in the applied force) while with ultrasonics the can was fully formed. The results are transferred to a graph of force vs neck diameter in figure 6.24 and here the graph of the theoretical forming force is also included (as on the earlier force - diameter graph). In this case the measured forming force (with ultrasonics) follows the theoretical curve much further, almost to the end of the forming process. Comparison with the measurements made in the early stages of the project shows that developments in the die technology have led to a further reduction in forming force but that the forming force has not been reduced below the theoretical zero friction force. This is strong circumstantial evidence in favour of the friction reduction theory of force reduction (see section 1.2).
INITIAL PEAK — CAUSED BY ACCELERATION OF TOOLING FROM REST

FINAL PEAK — CAUSED BY TOOLING HITTING END STOP SO THE FULL FORCE OF THE HYDRAULIC CYLINDER IS APPLIED

CALIBRATION CHECK — DIFFERENCE BETWEEN TRACES WITH / WITHOUT PLUNGER IS 1.4 DIV IE 2.9 kN 90 PSI IN 3" DIA CYLINDER GIVES 636 LBF = 2.8 kN

FORMING FORCE FOR CAN WITHOUT ULTRASONICS RISES TO PEAK 7.4 kN (3.5 DIV) AND THEN FALLS AS CAN IS CRUSHED

FORMING FORCE FOR CAN WITH ULTRASONICS RISES TO PEAK 6.5 kN (3.1 DIV) (CAN NOT CRUSHED)

PRESSURE STABILIZES WHEN TOOLING IS MOVING AT CONSTANT SPEED

FORCE WITH PLUNGER — AIR BUT NO CAN — BASE LINE FOR FORMING FORCE MEASUREMENT

FORCE WITH NO CAN AND NO PLUNGER — ZERO LEVEL

THE EFFECT OF ULTRASONICS IS A REDUCTION IN FORMING FORCE OF APPROX 2 kN (FAIRLY CONSTANT)

FIGURE 6.23 — FORMING FORCES MEASURED ON RIG
ESTIMATED STRENGTH OF CAN = 7.1 kN

THEORETICAL RELATIONSHIP ASSUMING ZERO FRICTION (UPPER AND LOWER BOUNDS)

KEY

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FIGURE 6.24 - MEASURED FORMING FORCE VS DIAMETER (LATER WORK)
6.6 CONCLUSIONS

Various methods have been used for evaluation of the die vibrations. ESPI (electronic speckle pattern interferometry) is potentially the most useful, offering full three-dimensional analysis of all visible surfaces of the die, even while forming a can. With further development the disadvantages of this system (cost, setting up and interpretation of results) could be reduced in the future. For evaluation of resonance modes and frequencies the admittance plotter (section 6.2.2) was used most. This gave convenient, accurate and revealing measurements of each die's performance.

Problems of "mode-switching" were found in dies with unwanted resonances close to the working frequency. The use of shaped dies was shown to overcome this problem.

Comparative measurements on dies with and without the tubular tuned mounting showed the improvements obtained by developing the method of attachment and the mounting geometry. The result of this development is a mounting system which accurately locates the die while allowing it to vibrate with minimal loss of energy.

Measurements of the force required to form a can showed that the effect of the ultrasonic vibrations was to reduce forming force by 40 to 60%. Where high reductions are required this may mean the difference between successfully necking the can and crushing it. The reduced forming force corresponded very closely to the theoretical zero-friction force, suggesting that the observed effect of ultrasonics may be caused by reduction or elimination of friction.
The majority of the work described here has been concerned with developing a practical system for can-forming with the assistance of ultrasonics. In this chapter further work is proposed with the aim of improving the understanding of the process and further developing the equipment.
7. Recommendations for further work

7.1 FURTHER STUDIES

The work described in previous chapters has led to the design of new ultrasonic tooling for forming metal cans, and some understanding of the novel forming process has been gained, but there is much more that could be done in this area. Industrial pressures to produce a practical system have ensured that development of the equipment has taken priority over theoretical studies aimed at gaining a fuller understanding of the underlying processes. The following sections discuss some studies which have not been attempted but which should produce valuable insights into the nature of the ultrasonic forming process.

7.1.1 Effects of ultrasonics

The beneficial effect of using ultrasonics can be attributed to a reduction in the forming force required to achieve the required reduction in can diameter. The results described in section 6.5 confirm this. Some explanation of the reasons for the observed force reduction has also been attempted and friction reduction proposed as the most likely mechanism, but the evidence is rather circumstantial and the other likely mechanism - swaging - may account for some or all of the effect. A new trial capable of differentiating between these proposed mechanisms, would greatly assist in the understanding of this process.

In order to achieve this the new trial would need to separate the two mechanisms while retaining the character of the real forming process. Experiments to measure friction coefficient have been conducted, eg. by Polanski et al [17] and Pohlman & Lehfeldt [16]. A similar experiment duplicating the normal force calculated for this process (the calculation itself assuming that friction reduction is the important mechanism) could demonstrate whether friction reduction to near zero could be achieved under these conditions. To remove the effects of metal-forming (which would
otherwise retain the possibility of a swaging mechanism) the can could be "unwrapped" and the ultrasonics applied perpendicular to a flat surface. The ultrasonic system would closely resemble a system for plastic welding, and such equipment would be well suited to this experiment.

The second part of this investigation would involve eliminating friction effects from the forming process. This might be done by fixing the axial position of the can while the ultrasonics are operating, and advancing the can only during short periods with the ultrasonics switched off. If the swaging theory is correct then for the period with ultrasonics applied the can should undergo a small plastic deformation, and after switching off this should permit the can to be advanced a short distance into the die at less than the usual forming force. The difficulty here would be in making the force application rigid enough to fix the can so that it does not advance into the die while the ultrasonics are working since this would permit the effects of friction reduction.

7.1.2 Effects of process on die vibrations

It has been observed that mode-switching (described in section 3.2) often occurs only when the die is loaded by the process of forming a can, which is of course precisely the time when the die needs to be working properly. Mode coupling is related to damping so this is also most likely during can forming.

Measurements to prove this have been difficult to obtain because the systems of measurement used worked best at low amplitude and / or in the absence of equipment for applying the forces necessary for forming. Specifically the measurements of Lucas [114], [115] using accelerometers were limited to small die amplitudes (the full amplitude of 10 μ at 20 kHz gives accelerations of approx. 15000 g which is enough to dislodge glued-on accelerometers) while ESPI [118], [119] measurements required an unobstructed view of the die face. Measurements with the Telsonic amplitude meter [115] were made during actual forming but at a single point only on the die.
To better understand the die behaviour under realistic conditions these measurements should be taken further. Modal analysis using non-contact sensors (like the Telsonic amplitude meter) at several points on the die could establish the levels of two or more modes of vibration, and the changes in these levels during forming. Recent developments in ESPI have also improved this process to permit direct measurement of high-amplitude vibrations and even direct measurement of material strain [121]. The use of special equipment to apply the forming forces from behind the die (see figure 7.01) would permit can-forming with minimal obstruction of the view of the die face.
7. Recommendations for further work

In parallel with these measurements a programme of analysis using simplified finite element models would assist the understanding of the modal behaviour. The forming process could be simply modelled as a mass and damper coupled to the inside surface of the die, and possible values of mass and damping could be found by adjusting the model to match the observed changes in power input and resonant frequency. To evaluate power input a dynamic analysis (full or reduced) would be required - see section 2.2. A constant amplitude forcing would approximate the "power by demand" control system of the ultrasonic generator. For a more accurate model of the process an approach similar to that of Kwak [122] (studying the "virtual mass" effects of water immersion on a vibrating system) could be employed.

One further area of study is the ultrasonic generator itself. The phenomenon of mode switching has been observed only in one of the ultrasonic systems tested, although mode coupling is expected to be independent of the system used. A theoretical study of the control philosophies employed by the ultrasonic generators might reveal the reasons for their very different performance.

The ultimate aim of all of this work would be to improve the understanding of the die vibrations and hence to design a better die.

7.1.3 Finite Element Analysis

Since this work was done advances in computer hardware have made far greater computing power easily affordable. A typical desktop 486 PC is now roughly equivalent in terms of computing power to the multi-user MicroVAX on which most of the finite element work was done. Software has also advanced, although developments have been mainly in the areas of user-friendliness rather than technical ability. Increasing competition from a variety of small software producers has also reduced the cost of finite element programs.
7. Recommendations for further work

The result of all this is that far more accurate and complete finite element models are now practical. In particular, three-dimensional models of the complete die can now be analysed much more easily to the required accuracy. This offers the possibility of obtaining a better understanding of interactions between modes, which can be difficult or impossible to analyse using simplified 2-D models. Furthermore the transducer might be included in the model, and using special piezo-electric elements even the electrical forcing function. The result of a fuller understanding of the vibrating system should be improved die design, possibly including radical changes not yet imagined.

Another area of rapid development has been computer graphics. Using readily-available hardware most finite element packages will now produce animated pictures of the modes of vibration with excellent image quality. These can greatly improve the communication of vibration modes, and issues like mode coupling could be understood much more easily.
7.2 NEW DESIGNS FOR ULTRASONIC TOOLING

As new applications are found for the ultrasonic forming process, new tooling will also be required. This section discusses the basis for new designs which may be required. Possible improvements to the current dies are also described with an indication of how they might be achieved.

7.2.1 New die designs

Apart from theoretical considerations of die performance there is another factor which controls die design - the product. This dictates the shape of the forming surface, the process requirement (eg. vibration amplitude) and the necessary material properties (hardness, corrosion-resistance etc.). Note that the benefit observed as a result of using ultrasonics has been limited to a reduction in forming force and this will generally only lead to economically viable applications where the force which can be applied is limited by some physical constraint - typically the strength of the product being formed. If this is not the case then the cost of ultrasonic equipment will not be justifiable in comparison with the cost of simply uprating the equipment applying the force.

A new die design will be required for each new forming process as the need arises. The design procedure described in chapter 4 is suitable for new dies to be manufactured from certain materials and within certain limits of product diameter. For new dies not falling within these limits (requiring different materials or geometry) the die design must be repeated from the start ie. trial-and-error finite element analysis to find a material / geometry combination which offers the required ultrasonic performance.

One example which may well be required is a die with a large inside diameter, for forming large diameter cans. The limit on inside diameter for the type of dies used to date is about 70 mm. This is because for a die to resonate in the RO mode as the inside diameter increases the outside diameter must decrease, and this must lead to a natural limit when the die
becomes too thin-walled. New materials (in particular the ceramic reinforced-aluminium composites - see section 1.3.5) may permit this limit to be raised slightly due to a higher sound velocity, but to overcome this limitation altogether a new mode of vibration must be used. The most likely candidate would be the 1RO mode (see section 3.1) which features motion similar to the R0 mode at its centre but with the outside of the die moving radially in antiphase and a nodal ring (figure 7.02).
7. Recommendations for further work

While this mode certainly exists and could be used for forming applications it will present more problems with unwanted modes of vibration because there are likely to be many more of these close to the working frequency. Tables 7.01 and 7.02 show for comparison the natural frequencies of a 20 kHz steel die of inside diameter 40 mm (a typical R0 mode die) and one of inside diameter 100 mm (tuned to the 1R0 mode).

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Table 7.01 - Natural frequencies (kHz) of a hollow steel cylinder inside Ø40, outside Ø140, thickness 50 mm (Ansys predictions)
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Table 7.02 - Natural frequencies (kHz) of a hollow steel cylinder inside $\Omega 100$, outside $\Omega 413$, thickness 50 mm (Ansys predictions)
7. Recommendations for further work

7.2.2 New ultrasonic mountings

The tubular mounting design developed for this project has proved extremely efficient and robust. The only criticism which could be levelled at it is the cost, which can be high particularly for a die and mounting made together from solid titanium alloy. The material cost of a billet of Ti 6-Al 4-V, suitable for a typical die and mounting, is approximately £500, and the machining cost is of the same order (most of the billet must be machined away). If cost is an important factor then an aluminium mounting may be used, shrink-fitted to the die, although the power losses from a system of this type will probably be higher.

In some applications, however, the tubular mounting system may not be suitable - particularly where space is limited. The contour plots of power losses (figures 5.19 - 5.22) discussed in chapter 5 show little scope for reducing the length of the mounting tube below about 50 mm, although the “tail” may be foregone. Further reduction in the mounting length could only be achieved by a radical change to the design.

Another possible requirement of future mountings is a larger internal diameter (eg. to accommodate larger products). This also would require a radical redesign. Note that while the mounting remains a thin tube the 1R0 mode (potentially of use for large dies) effectively does not exist. In practice this is probably not a serious limitation because this type of die would have a circular nodal line at which its radial amplitude would be zero, and this could be a suitable point for a rigid fixing. Note, however, that a node of radial motion will often be an antinode of motion in other directions (eg. axial) and this could limit the efficiency of such a mounting.

New mounting systems in any number of different forms may be developed to suit new applications as these arise. The general principles used for the design and optimisation of the tubular mounting should be used as a basis for any new design.
Ultrasonic motors convert vibrations to motion (rotary or linear). This may be achieved either by creating vibrations at an angle to the surface or by combining two vibration modes to generate elliptical motion at the surface (figure 7.03). Angled vibrations (for example used by Schoenwald et al. [66]) rely on the surfaces separating with the rotor being thrown forward each time it loses contact with the stator. This is a simple technique but the speed varies greatly with load and efficiency is low. The technique using two vibration modes can offer much higher efficiency and better speed control. In this process (used for example by LeLetty [67]) two standing waves are set up with a 90° phase angle so that the combined motion simulates a travelling wave. This allows for continuous contact between the stator and the rotor with no slip provided that the maximum torque is not exceeded.
If the ultrasonic die could be made to act like a motor, dragging the can in, then it could be much more effective than the current dies which, at best, may allow the can to be pushed in without frictional resistance. As for the ultrasonic motors, this could possibly be achieved in two ways:

Generating a surface motion on the inside surface angled towards the back of the die would not be too difficult. "Unbalancing" the die by machining out the back face and placing the forming profile towards the back of the die could achieve this type of motion. Figure 7.04 shows an Ansys analysis of a simple die of this type. This is an axisymmetric model so half of the cross-section is shown. The centreline is along the y-axis. Note the direction of motion on the "forming surface" ie. the sloping inside edge of the model. One major disadvantage of this die would be the nature of the transducer motion - the outside surface of the die does not have the desired uniform radial motion. Furthermore as discussed earlier this type of motor is not ideal for use under
high loading and it is likely that it would have little more effect than the standard radial-mode ultrasonic dies.

Generating an elliptical surface motion could be much more effective in reducing the forming force, because the force dragging the can in would increase in proportion to the radial force. Unfortunately this type of motion is also much more difficult to achieve. In the motor applications a ring is used and the two standing waves are set up in different angular positions on the ring. Ideally, for a perfectly uniform ring, the resonant frequency should be independent of angular position, so all resonances should be at the same frequency. Even allowing for imperfections the difference between the resonant frequencies at different positions should be small enough to allow the use of a single driving frequency, which is essential to maintain the constant phase angle required to produce a constant velocity output. For the ultrasonic die it is much more difficult to match the resonant frequencies in this way. The required effect (elliptical surface motion dragging the can into the die) implies a combination of radial and axial motion on the inside of the die. The two most common modes which produce this type of motion are the radial and torsional axisymmetric modes (see section 3.1) but these rarely if ever appear at the same natural frequency. In order to make this proposal work, therefore, it would be necessary first to produce a die shape for which the two modes appeared at the same natural frequency and second to develop a method of driving the two modes at the same frequency with the required phase angle.
CONCLUSIONS

Finite Element Analysis

Finite element analysis has been used extensively for the design of ultrasonic tools. Several different types of finite element model have been used to suit the needs of each specific application.

Hardware limitations have required the development of highly efficient models which nevertheless give accurate results for specific performance data. Methods of simplifying the analysis which have been considered include the use of reduced dynamic analysis, two-dimensional analysis and partial models. The advantages and limitations of each option have been investigated.

Accuracy of results has been studied and a set of recommendations prepared. Excellent accuracy has been shown to be achievable provided the appropriate guidelines are followed.

Parametric analysis has been used to determine and display the effects of changing the geometry and material properties of the ultrasonic tooling.

Example program listings are given in appendix 3 to demonstrate the use of these techniques.
Design of Ultrasonic Tooling

The analysis described above has been used to gain a much fuller understanding of the modes of vibration of ultrasonic dies and the reasons for undesirable behaviour sometimes observed in practice. Much of this is associated with unwanted modes of vibration of the dies, some of which are quite unexpected. A comprehensive system of mode nomenclature has therefore been defined. By consideration of the forming process and study of relevant literature the ideal "working" mode of vibration has been selected. The design of ultrasonic dies to promote this mode to the exclusion of others has then been demonstrated with reference to theoretical and practical problems encountered.

Study of materials has identified some which are particularly suitable for use with ultrasonics, offering low acoustic losses. The stress analysis of dies, to ensure acceptable fatigue life, has been described in detail. Practical considerations for ease of machining and assembly are also discussed.

Highly efficient piezo-electric transducers, rarely used before in metal-forming, have been shown to be well suited to this application, provided that the guidelines developed for die design are followed.

Novel "shaped dies" have been developed to overcome special problems in certain dies. The advantages of these dies have been demonstrated, and a patent covering this work has been filed [124].

After studying known methods of mounting ultrasonic tools, a novel mounting system offering high-accuracy and low losses has been designed and optimised.

Combining the developments listed above, highly efficient ultrasonic systems have been produced for the necking of steel aerosol cans. Detailed procedures have been prepared for the manufacture of these dies and mountings.
For new dies similar in concept to the existing ones, a simplified design procedure has been prepared, based on the results of many simple analyses, to permit approximate checking and optimisation of die designs without further finite element analysis. A full set of design data and detailed instructions for use are included.

Verification of results

Many of the predictions of the finite element analysis have been verified by measurements of real dies. This involved first establishing testing methods for determination of resonant mode and frequency, and then evaluating the properties of each die and mounting. Detailed lists of measurements have been prepared along with discussion of the important points. The performance of the new designs has been shown to match expectations very closely. Where significant differences have been noted, and particularly where these differences are consistent for several different dies, these have been clearly identified and their implications discussed.

One important measuring technique used on this project was Electronic Speckle Pattern Interferometry (ESPI). To assist in the identification of modes in real time, the finite element results were processed to produce a theoretical simulation of the ESPI output, for comparison with the measured results. This proved valuable not only in identifying modes of vibration but also in understanding the irregularities of die motion measured by this technique.

The most important test of ultrasonic die performance was measurement of the force required to deform the can, since this is ultimately the reason for the use of ultrasonics. These trials showed that the effect of the ultrasonic vibrations was to reduce forming force by 30 to 60%. Where high reductions are required this may mean the difference between successfully necking the can and crushing it. Comparison of measurements made during the early
stages of the project with those made later shows the benefits of improvements to the ultrasonic tooling. For later results the reduced forming force corresponded very closely to the theoretical zero-friction force, suggesting that the observed effect of ultrasonics may be caused by reduction or elimination of friction.

**Project aims**

The aim of the project was to develop a better understanding of the process of ultrasonic necking and hence to improve its effectiveness. This has been achieved through the application of new equipment, materials and designs.
APPENDIX 1

THEORETICAL ANALYSIS OF NECK FORMING PROCESS

The aim of this analysis is to predict the force required to form the can under ideal conditions and to improve the understanding of which parameters affect the forming force.

The principle of energy conservation is used, deriving the forming force from the work done deforming the material. Other sources of energy loss (friction, material heating) have been ignored in order to simplify the analysis and find the limits of the process under ideal conditions.

One major unknown is the radial stress in the material ($\sigma_r$). In theory there is always a clearance between the tools which is greater than the material thickness so the radial stress should be zero on the inside surface of the can. In reality it is known that the can does contact the internal plug (if it did not there would be no reason to use the plug) so there must be some radial stress, at least locally. To overcome this problem the analysis is performed both for a high radial stress and zero radial stress giving upper and lower bounds on the real situation.

Using the Tresca yield criterion, yield in the hoop direction can happen only when the difference in stress between the hoop stress $\sigma_h$ and one of the orthogonal stresses (longitudinal $\sigma_l$ or radial $\sigma_r$) is equal to the yield stress $\sigma_y$, ie.

$$\sigma_y = \sigma_h - \sigma_r$$, or

$$\sigma_y = \sigma_h - \sigma_l$$, whichever is the greater.
Note that all stresses are compressive. For convenience compressive stresses will be shown positive. If the radial stress $\sigma_r$ is zero then the first equation applies and:

$$\sigma_h = \sigma_y \quad \text{(lower bound on stress)} \quad \ldots \quad (1)$$

Conversely, if there is a compressive stress in the radial direction of greater magnitude than the longitudinal stress then the second equation applies and:

$$\sigma_h = \sigma_y + \sigma_r \quad \text{(upper bound on stress)} \quad \ldots \quad (2)$$

Consider the can part-way through the necking process. The diameter at the end of the neck is $r_m$. The material is increasingly thickened and work hardened as the diameter is reduced. The analysis is based on a small element of length $l$ at a radius $r$ where the yield stress is $\sigma_y$. In the undeformed cylindrical part of the can these element dimensions will be called $l_0$, $r_0$, $\sigma_{y0}$ and $t_0$ respectively.

Assume that the can material work hardens linearly with plastic strain. Then it can be described by the following formula:

$$\sigma_y = \sigma_{y0} \left[ 1 + \frac{\epsilon_p(r_0-r)}{r_0} \right] \quad \ldots \quad (3)$$

The aim of the analysis is to calculate the total work done advancing the can a short distance into the die. First calculate the work done reducing the radius of our ring element from $r$ to $(r-dr)$. This is the hoop force (stress x area) multiplied by the hoop displacement:

$$dW = \sigma_h \cdot l \cdot t \cdot 2\pi \cdot dr \quad \ldots \quad (4)$$

By conservation of volume within the element,

$$r \cdot l \cdot t = r_0 \cdot l_0 \cdot t_0 \quad \ldots \quad (5)$$

And substituting (5) into (4) gives:

$$dW = 2\pi r_0 l_0 t_0 \cdot \sigma_{h-1} \cdot \frac{dr}{r}$$
Integrating this with respect to \( r \) gives the total work done advancing the distance \( l \):

\[
W = \int_{r_m}^{r_0} 2\pi r_0 t_0 \sigma_h \left( \frac{1}{r} \right) dr
\]

or

\[
W = \int_{r_m}^{r_0} 2\pi r_0 t_0 \sigma_h \cdot dr
\]

The force which must be applied to the can to provide this work is:

\[
F = \frac{W}{l_0}
\]

or

\[
F = \int_{r_m}^{r_0} 2\pi r_0 t_0 \sigma_h \cdot dr
\] .......(6)

Now substitute for \( \sigma_h \) using the upper and lower bound values calculated earlier. First the lower bound using (1):

\[
F = \int_{r_m}^{r_0} 2\pi r_0 t_0 \sigma_y \cdot dr
\]

and substituting for \( \sigma_y \) from (3):

\[
F = \int_{r_m}^{r_0} 2\pi r_0 t_0 \sigma_y \left[ \frac{1 + c_0(\sigma_0 - r)}{r} \right] dr
\]

\[
F = \int_{r_m}^{r_0} 2\pi r_0 t_0 \sigma_y \left[ 1 + c_0 \right] \cdot dr
\]
Integrating this gives:

$$F = 2\pi r_0 t_0 \sigma_y \left[ (1 + c) \log_e \left( \frac{r_0}{r_m} \right) - c \left( r_0 - r_m \right) \right]$$

The constant term here ($2\pi r_0 t_0 \sigma_y$) is the force at which the cylindrical part of the can will yield under axial compression. Call this $F_y$. Then:

$$F_1 = F_y \left[ (1 + c) \log_e \left( \frac{r_0}{r_m} \right) - c \left( r_0 - r_m \right) \right]$$

This lower-bound force will be called $F_1$.

The upper bound on forming force is calculated in a similar way but this case is slightly more complicated because the longitudinal stress $\sigma_l$ (which varies around the neck) is also involved. Substituting (2) into (4) gives:

$$dW = 2\pi r t \sigma_y$$

Now $\sigma_l$ is the longitudinal force $F$ divided by the cross-sectional area, so:

$$dW = 2\pi r t (\sigma_y + \sigma_l)$$

Also $dW = l \cdot dF$, giving:

$$dF = 2\pi r t \sigma_y + \sigma_l$$

Dividing through by $r$ gives a form which can be conveniently integrated:

$$\frac{1}{r} \cdot dF + F \cdot \left( -\frac{1}{r} \right) = 2\pi \sigma_y$$

Comparing this with the standard form $u \cdot dv + v \cdot du = d \cdot (u, v)$

$$dx \quad dx \quad dx$$
allows the above to be integrated as:

\[ F = \int_{r}^{2\pi r} \frac{\pi t \sigma_y}{r} dr \]

\[ F = 2\pi r \int_{r}^{\frac{r}{2\pi r}} \frac{\sigma_y}{r} dr \]

Then using (5) (conservation of volume within the element),

\[ F = 2\pi r \int_{r}^{\frac{r}{2\pi r}} \sigma_y \cdot \frac{l_0 l_0}{r} dr \]

This expression remains impossible to solve without knowledge of how the material thickens and lengthens during forming, but it can be simplified by assuming that \( l = l_0 \). In reality the longitudinal length of the can always increases during forming (known from measurement of formed cans) so \( l > l_0 \).

This means that by taking \( l \) equal to \( l_0 \) will give a value for \( F \) which is greater than the true value. This means that the value obtained will still be an upper bound on the real forming force. Taking \( l = l_0 \) gives:

\[ F = 2\pi r \int_{r}^{\frac{r}{2\pi r}} \sigma_y \cdot \frac{l_0}{r^2} dr \]

Substituting for \( \sigma_y \) from equation (3) gives:

\[ F = 2\pi r \int_{r}^{\frac{r}{2\pi r}} \sigma_y \cdot \frac{l_0}{r^2} \cdot \left( 1 + c \frac{r_0}{r} \right) dr \]

Substituting for the yield force \( F_y \) as before and tidying gives:

\[ F = F_y r \int_{r}^{\frac{r}{2\pi r}} \left( 1 + c \right) - \frac{c}{r} \cdot \frac{r_0}{r} dr \]
This equation can be integrated:

\[ F = F_y \cdot r \left[ - (1+c) - \frac{c}{r} \cdot \log_0(r) + B \right] \]

(where \( B \) is the constant of integration. Knowing that longitudinal force is zero at the minimum radius \( r_m \) allows solution for \( B \):

\[ B = \frac{(1+c) + \log_0(r_m)}{r_m/r_0} \]

Also we are interested only in the force in the cylindrical part of the can (i.e. the maximum longitudinal force) where \( r = r_0 \).

So the upper limit on \( F \) (\( F_u \)) becomes:

\[ F_u = F_y \left[ (1+c) \cdot (r_0 - r_m) + c \cdot \log_0(r_m/r_0) \right] \]

We now have expressions for \( F_1 \) and \( F_u \) the lower and upper bounds on the forming force. To evaluate these it is necessary to substitute for \( F_y \), \( c \) and \( r_0 \). The aim is to evaluate the expressions over a range of \( r_m \) up to the point of certain yield when \( F_1 = F_y \).

For the 45 mm aerosol cans used in this work, \( r = 22.5 \) mm and \( t = 0.21 \) mm.

By micro-hardness measurement of a number of cans averaged values were obtained for hardness initially and after necking to approx 30 mm diameter. These were converted (approximately) to the following yield stresses:

\[ \sigma_y = 240 \text{ MPa at } \phi 45. \]
\[ \sigma_y = 300 \text{ MPa at } \phi 30. \]

Substituting these values into equation (3) allows solution for \( c \), giving \( c = 0.75 \).

Using these values, \( F_y (2.\pi.r.t.\sigma_y) \) can be calculated:

\[ F_y = 7125 \text{ N} \]
And all of these numeric values can be substituted into the equations for $F_1$ and $F_u$, giving:

$$F_1 = 7125 \left[ 1.75 \log_{10} \left( \frac{22.5}{r_m} \right) - \frac{(22.5 - r_m)}{30} \right]$$

$$F_u = 7125 \left[ 1.75 \left( \frac{22.5 - r_m}{r_m} \right) + 0.75 \log_{10} \left( \frac{r_m}{22.5} \right) \right]$$

(The results will be obtained in newtons. Since the expressions in brackets are dimensionless it is convenient to evaluate them using millimetre units.)

It is these expressions which have been used to produce the graphs of theoretical load vs neck diameter shown in chapter 6 figures 6.21 and 6.24.
APPENDIX 2

ANALYSIS OF RADIAL VIBRATIONS

OF A HOLLOW CYLINDER

The aim of this analysis was to obtain some verification of the finite element results by a "classical" analysis of the die vibrations. To simplify the analysis plane stress is assumed. Initially the thickness of the die is assumed constant, although it will also be demonstrated that this work can be extended to the more general case of varying cross section (provided the change in cross section is neither too great nor too sudden) by a simple finite-difference technique. The main shortcoming of this analysis is that only the axisymmetric radial mode is covered. Analysis of the harmonic modes is much more complicated and in view of the availability of finite element tools this analysis was not justified.

Consider a hollow cylinder with inside radius $R_1$, outside radius $R_2$ and thickness $t$. Somewhere within it is a small element at radius $r$ with radial dimension $dr$ and tangential dimension $d\theta$.

The mass of the element is $\rho \cdot r \cdot dr \cdot d\theta \cdot t$

Given some displacement $u$ from the equilibrium position, with corresponding acceleration $\ddot{u}$, and components of stress $\sigma_r$ and $\sigma_\theta$ in the radial and tangential directions respectively, equilibrium gives:

$$\sigma_r dr d\theta + \sigma_\theta r d\theta + \rho \ddot{u} r dr d\theta = \left[ \sigma_r + \delta \sigma_r dr \right] (r + dr) d\theta$$

Ignoring second order terms:

$$\sigma_r dr d\theta + \rho \ddot{u} r dr d\theta = \sigma_r dr d\theta + r \delta \sigma_r dr d\theta$$

$$\delta r$$

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And simplifying:

\[ \sigma_r - \sigma_i = r \begin{bmatrix} p \ddot{u} - \frac{\delta \sigma_r}{\delta r} \\ \frac{\delta \sigma_i}{\delta r} \end{bmatrix} \]  

..... (1)

Elasticity links stress to strain. Assuming plane stress (i.e. stress through the thickness is zero) then:

\[ \varepsilon_r = \frac{1}{E} (\sigma_r - v \sigma_i) \]

and:

\[ \varepsilon_i = \frac{1}{E} (\sigma_i - v \sigma_r) \]

These can be written:

\[ \varepsilon_r + v \varepsilon_i = \frac{\sigma_r}{E} (1 - v^2) \]  

..... (2a)

and:

\[ v \varepsilon_i + \varepsilon_i = \frac{\sigma_i}{E} (1 - v^2) \]

E

Combining these:

\[ (\sigma_r - \sigma_i) = E \begin{bmatrix} \varepsilon_r + v \varepsilon_i - \sigma_r - \varepsilon_i \\ (1 - v^2) \end{bmatrix} \]

\[ (\sigma_r - \sigma_i) = E \begin{bmatrix} \varepsilon_r - \varepsilon_i \\ (1 + v) \end{bmatrix} \]  

..... (2b)

also differentiating (2a) with respect to \( r \) gives:

\[ \frac{\delta \sigma_r}{\delta r} = E \begin{bmatrix} \delta \varepsilon_r + v \delta \varepsilon_i \\ (1 - v^2) \end{bmatrix} \]  

..... (2c)

Compati\( \text{bility also gives the following:} \)

\[ \varepsilon_r = \frac{\delta u}{\delta r} \]  

..... (3a)
and:

\[ \varepsilon_1 = \frac{u}{r} \quad \ldots \quad (3b) \]

To solve the above equations of equilibrium, elasticity and compatibility, begin by substituting (3a) and (3b) in (2b):

\[ (\sigma_r - \sigma_1) = E \left( \frac{\delta u}{r} - u \right) \quad \ldots \quad (4a) \]

and in (2c):

\[ \begin{align*}
\frac{\delta \sigma_r}{\delta r} &= E \left[ \delta^2 u + \nu \left( r \frac{\delta u}{\delta r} - u \right) \right] \\
\frac{\delta \sigma_r}{\delta r} &= E \left[ \delta^2 u + \nu \frac{\delta u}{\delta r} \right] \quad \ldots \quad (4b)
\end{align*} \]

Next substitute (4a) and (4b) into (1):

\[ \frac{E}{(1 + \nu)} \left[ \delta u - u \right] = \rho r \ddot{u} - E \left. \left( \frac{\delta^2 u}{\delta r^2} + \nu \frac{\delta u}{\delta r} \right) \right) \]

\[ \left. \frac{\delta}{\delta r} \left[ \delta^2 u - u \right] \right) \quad \ldots \quad (4c) \]

This is the radial wave equation. To solve it, consider the displacement \( u \) as an unknown function of radius and time ie.

\[ u(r, t) = f(r) \cdot g(t) \]

(where \( f \) and \( g \) are the unknown functions). Differentiating gives:

\[ \frac{\delta u}{\delta r} = f'(r) \cdot g(t) \quad \frac{\delta u}{\delta r} \]
A2. Analysis of radial vibrations of a hollow cylinder

\[ \ddot{u} = f''(r) \cdot g(t) \]
\[ \ddot{u} = f(r) \cdot g''(t) \]

Also for simplicity use a constant \( k_1 \)

\[ k_1 = \frac{E}{\rho(1 - \nu^2)} \]

Substitute these into the wave equation (4c):

\[ f(r) \cdot g''(t) = k_1 \left[ f''(r) \cdot g(t) + f'(r) \cdot g(t) - f(r) \cdot g(t) \right] \]

Here we have something which is a function only of time, and a function only of radius. Hence it must be a constant. Call this constant \( k_2 \). Then:

\[ \frac{g''(t)}{k_1 \cdot g(t) \cdot f(r)} = \frac{1}{r} \left[ f''(r) + f'(r) - f(r) \right] \]

Extracting the time functions from this gives:

\[ g''(t) - k_1 \cdot k_2 \cdot g(t) = 0 \quad \text{...... (5a)} \]

While extracting the radius functions gives:

\[ f''(r) + f'(r) - f(r) \cdot \left[ k_2 + \frac{1}{r^2} \right] = 0 \quad \text{...... (5b)} \]

Equation (5a) has the standard solution for steady state harmonic oscillation:

\[ g = e^{(\pm \sqrt{k_2} \cdot t)} \]
Where the angular frequency $\omega$ is given by:

$$\omega^2 = -k_2 k_1$$

Hence:

$$k_2 = -\frac{\omega^2}{\omega^2 \rho(1 - \nu^2)}$$

For convenience, let us use a new constant $k$ where:

$$k^2 = -\frac{2\pi^2(1 - \nu^2)}{E}$$

So (5b) becomes:

$$f''(r) + \frac{f'(r)}{r} + f(r)\left[\frac{k^2}{r^2} - \frac{1}{r^2}\right] = 0$$

In this, substitute a variable $s$, where:

$$r = s, \quad f(r) = f(s), \quad f'(r) = f'(s), \quad f''(r) = k f''(s)$$

giving:

$$k f''(s) + \frac{f'(s)}{s} + f(s)\left[\frac{k^2}{s^2} - \frac{k^2}{s^2}\right] = 0$$

multiplying through by $s^2/k$ gives:

$$s^2 f''(s) + s f'(s) + (s^2 - 1)f(s) = 0$$

This is Bessel's equation (first order). The solution is:

$$f(s) = \alpha J_1(s) + \beta Y_1(s)$$

where $\alpha$ and $\beta$ are constants and $J_1$ and $Y_1$ are Bessel functions of order unity.

Differentiating this gives:

$$f'(s) = \alpha J_1'(s) + \beta Y_1'(s)$$
A2. Analysis of radial vibrations of a hollow cylinder

Full solution of the equations requires use of the boundary conditions, specifically the radial stress \( \sigma_r \), and amplitude \( f(r) \). From the elasticity equations:

\[
\sigma_r = E \cdot \left( \varepsilon_r + \nu \cdot \varepsilon_t \right)
\]

\[
(1 - \nu^2)
\]

\[
\sigma_r = \frac{E}{1 - \nu^2} \left[ \frac{df(r)}{dr} + \nu \cdot f(r) \right]
\]

\[
(1 - \nu^2) \int \frac{dr}{r}
\]

\[
\sigma_r = \frac{E}{1 - \nu^2} \left[ \frac{df(s)}{ds} + \nu \cdot f(s) \right]
\]

\[
(1 - \nu^2) \int \frac{ds}{s}
\]

\[
\sigma_r = \frac{E}{1 - \nu^2} \left[ \alpha \cdot J_1'(s) + \beta \cdot Y_1'(s) + \nu \left[ \alpha \cdot J_1(s) + \beta \cdot Y_1(s) \right] \right]
\]

\[
(1 - \nu^2) \int \frac{ds}{s}
\]

To simplify this equation we can introduce two new functions:

\[
I(s) = \frac{E}{(1 - \nu^2)} \left[ J_1'(s) + \nu \cdot J_1(s) \right]
\]

\[
(1 - \nu^2) \int \frac{ds}{s}
\]

and:

\[
X(s) = \frac{E}{(1 - \nu^2)} \left[ Y_1'(s) + \nu \cdot Y_1(s) \right]
\]

\[
(1 - \nu^2) \int \frac{ds}{s}
\]

giving:

\[
\sigma_r = \alpha \cdot I(s) + \beta \cdot X(s)
\]

\[
(6d)
\]

Now (6a) and (6d) are simultaneous equations which can be solved for \( \alpha \) and \( \beta \), given the boundary conditions \( \sigma_r \) and \( f(s) \), as follows:
A2. Analysis of radial vibrations of a hollow cylinder

\[ \alpha = \frac{\sigma_r Y_1(s) - f(s) X(s)}{I(s) Y_1(s) - J_1(s) X(s)} \quad \ldots \quad (6e) \]

\[ \beta = \frac{\sigma_r J_1(s) - f(s) I(s)}{J_1(s) X(s) - I(s) Y_1(s)} \quad \ldots \quad (6f) \]

All the required equations are now available for evaluation of the motion of a hollow cylinder vibrating at a chosen frequency. The procedure is as follows:

1) Calculate \( k \) from the material properties and chosen frequency using equation (5c).

2) Calculate the dimensionless radius \( s \) corresponding to the inside radius \( R_1 \).

3) Evaluate the Bessel functions \( J_1, Y_1, J_1', \) and \( Y_1' \) using tables or a series solution for this value of \( s \).

4) Evaluate the extra functions \( X_1 \) and \( I_1 \).

5) On the inside surface of the cylinder the amplitude can be given some nominal value. This is the value of \( f(r) \). \( f(s) \) can be calculated from this. Furthermore the radial stress is zero (because the surface must be free of normal stress). Substituting for \( \sigma_r \) and \( f(s) \) in equations (6e) and (6f) gives \( \alpha \) and \( \beta \).

6) For the outside radius \( R_2 \) calculate the corresponding values of \( s, J_1, Y_1, J_1', Y_1', X_1 \) and \( I_1 \).

7) Using these values plus the previously calculated constants \( \alpha \) and \( \beta \) in equations (6a) and (6d) to evaluate the stress and amplitude on the outside surface of the cylinder.

Note that free vibration at the chosen frequency is possible only if the radial stress on the outside surface evaluates to zero. By repeating steps 6 and 7 for different values of \( R_2 \) a suitable value may be found (this type of iterative procedure is practical only once the calculations have been automated).
One limitation with this analysis is that only a simple hollow cylinder, made of a single material, can be analysed. The analysis can be extended, however, to cover a shaped cross section and a number of materials by using a finite-difference technique, analysing a series of concentric thin rings. Starting on the inside surface, the radial amplitude and stress on the outside of the first ring are calculated as above. The amplitude must be transmitted directly to the next ring, while the stress will be transmitted to the next ring according to the ratio of their areas (so that applied forces are equal and opposite). Thus new values are available for the boundary conditions on the inside of the next ring, which can be analysed in the same way as the first, and so on. The analysis of each ring is separate so different materials and different thicknesses can be accommodated.
APPENDIX 3

EXAMPLE PROGRAM LISTINGS

The program listings reproduced here are examples of typical Ansys analyses. These commands could be typed in by hand (using Ansys interactively) or read in automatically. For each example, a general description of the function of the program is given first, followed by the command listing.

Some of the example listings are Ansys command files while others are VAX command files which include the command to run the Ansys program and thereafter an embedded Ansys command-file. VAX system commands are preceded by the dollar symbol "$" as the first character on a line while Ansys commands have no such identifier.

Comment lines have been included in the listings to assist understanding of the command lines. Note that comments are included only to indicate the purpose of the commands. They are ignored by both the VAX system and the Ansys program. VAX system comment lines are preceded by an exclamation mark so the line starts "$!". The Ansys program permits two conventions for comment lines. Either the symbols "C****" are placed at the beginning of the line and the whole line is ignored, or the symbols " **" are placed after a command and the remainder of the line is ignored. (Note that in this case the space before the asterisk is essential - without it the program would treat the asterisk as a multiplication symbol.)

Note also that some quite complicated techniques have been used in these listings to overcome limitations in the Ansys program. Later versions of the program provide extra functions which make many of these techniques unnecessary.
A3.1 Static Analysis Listing

This listing demonstrates the use of static analysis to predict the stresses and distorted shape of a two-part die with an interference fit. Axi-symmetric harmonic 8-noded elements (STIF83) have been used. Normal axi-symmetric elements could have been used but the harmonic elements were used to minimise changes from the modal analysis version of the program.

Note the use of parameter INTR to specify the radial interference which can thus be easily changed. The geometry definition in this example is simple but long-winded. Nodes and elements are defined directly, rather than using automatic meshing. This is convenient because the constraint equations (which define the interference fit) require the explicit use of node numbers. (Using automatic meshing the user generally does not know in advance which node numbers will be assigned.)

C----------START OF EXAMPLE 1 ----------
C
IPREP7
C Enter general-purpose pre-processor PREP7
/TITL,NK,AEROSOL DIE - INTERFERENCE FIT STRESS
C Specify analysis title
C
C First set analysis options
KAN,0 Analysis type 0 - static
ET,1,83 Element type STIF83 (Axi-harmonic)
INTR,50E-6 Parameter INTR will specify radial interference (50u is heavy press fit)
C
C Next specify material properties
EX,1,1260E9 Youngs Modulus
DENS,1,7300 Density for Nikro 292
NUXY,1,.27 Poissons ratio
C
EX,2,74E9 Youngs modulus
DENS,2,2800 Density for Aluminium
NUXY,2,.34 Poisson’s ratio
C
C Now specify geometry, first nodes...
N,1,.0155,.003 node 1 at x=0.155, y=0.003
N,3,.0155,.021 similarly for node 3
FILL,1,3 fill in (node 2) between 1 and 3
N,4,.0225,.005
N,8,.0225,.044
FILL,4,6
N,7,.027,.003
N,10,.027,.024
FILL,7,10
N,12,.027,.044
FILL,10,12 (nodes 1 to 12 form outline of insert)
C
NGEN2,100,7,12,1,INTR
C This command generates another set of nodes numbered
107 to 112 at a distance INTR outside corresponding nodes 7 to 12.
C
N,13,.036
N,14,.034,01
N,15,.034,0378
N,16,.036,046
N,17,.032
N,18,.042,016
N,19,.042,0318
N,20,.050,046
N,21,.065
N,24,.065,046
N,25,.0845
N,28,.0845,046
FILL,21,24
FILL,25,28 (nodes 3 to 28 form die outer)
C
C Next define element connectivity
MAT,1
E,1,2,108 107 using -aterial 1 (Nikro 292)...
ENGEN,5 ALL and generate 4 more (5 total) from it.
C
MAT,2
E,7,8,14,13 now use material 2 (Aluminium L168)...
E,9,18,14 and specify all elements of die outer.
E,10,10,19,18 (note the element formation in this part...
E,10,11,16,15 of the die outer does not lend itself to...
E,11,12,16,15 automatic element generation as above)
E,13,14,18,17
E,15,16,20,19
E,17,18,22,21 (note outer 5 elements from one...
EGEN3,1,1
EGEN2,4,3
C
C Now speciy node coupling...
CE,1,INTR7,UX,1,107,UX,-1
C This specifies a constraint equation linking node 7 to node...
C 107. (Radial motion of 7)=Radial motion of 107)+INTR
CP1,1,1,1 repeats the above command 5 more times (6 in total)
C incrementing the equation number and the 2 node numbers

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A3. Example program listings

C*** Example program listings

C*** each time so linking 8 to 108 in equation 2 and so on up to
C*** 12-112 in equation 6.
C***
CP,1,UY,7,107
C*** This couples the axial (UY) motion of node 7 to that of
C*** node 107 (effectively joining them in the axial direction).
RP,1,1,11 **and repeat for the other 5 pairs of nodes.
C***
C*** Geometry is now fully defined. Note that no external forces
C*** or constraints will be applied - the displacements and
C*** stresses are generated internally by the interference.
C***
I,PNUM,MAT,1
EPLO
C*** Finally, write the analysis file...
AFWR
C*** and leave the pre-processor...
FINI
C***
C*** Back at 'Routine Begin' level, run the analysis...
/IEXE  *Specify execute option (rather than check-run)
/ANP,27 *Transfer control to analysis file (FILE27)
FINI  *Finish analysis
C***
C*** Analysis complete. Results stored in FILE12 can be
C*** examined using the postprocessor POST1.
C***
C******************************** END OF EXAMPLE 1

A3.2 Modal Analysis Listing

This demonstrates the use of modal analysis to predict stresses generated by vibration in the radial axisymmetric (RO) mode. The parameter AMPL is used to define the vibration amplitude. In this case automatic meshing is used to reduce the work required defining nodes and elements and to provide the flexibility to change mesh density. "Seismic excitation" is used to drive the die in the radial direction. This specifies a radial amplitude at which the die will vibrate so that stresses can be calculated for each mode (normally in modal analysis the amplitude is arbitrary so any stress value calculated is also arbitrary)

C******** START OF EXAMPLE 2 **********************
C***
C*** Enter general-purpose pre-processor PREP7
C***
/TITL,NK /AL AEROSOL DIE - RO MODE STRESS
C*** Specify analysis title
C***
C*** First set analysis options...
KAN,2 *Analysis type 2 - modal
KAY,2,-1 *Specify modes by frequency range
EXTM,100,30000 *Frequency range of modes to extract
EXMO,100,30000 *Frequency range of modes to expand
TOTAL 200 *Total number of master degrees of
C***
freedom
ET,1,83 *Element type STIF83 (Anti-harmonic)
INTR=50E-6 *Parameter INTR is used for geometry
C***
*definition but interference not used
*here.
AMPL=10E-6 *Parameter AMPL will specify radial
C***
*amplitude of vibration
C***
C*** Next specify material properties...
EX,1,260E9 *Young's Modulus
DENS,1,2300 *Density ) for Nikro 292
NUXY,1,27 *Poisson's ratio)
C***
EX,2,74E9 *Young's modulus)
DENS,2,2800 *Density ) for Aluminium
NUXY,2,34 *Poisson's ratio ) L168
C***
C*** Now specify geometry, first nodes
N 1 0155, 003  *node 1 at x= 0155 y= 003
N 9, 0155, 021  *similarly for node 3
FILL,1.3  *fill in (node 2) between 1 arc 3
N 4 0225, 025
N 6 0225, 044
FILL,4.6
N 7, 027, 003
N 10, 027, 024
FILL,7,10
N 12, 027, 044
FILL,10.12  *nodes 1 to 12 form outline of inser
C***
NGEN,2,100,7,12,1,INTR
C*** This command generates another set of nodes numbered
107 to
C*** 112 at a distance INTR outside corresponding nodes 7 to
12
N 13, 036
N 14, 034, 01
N 15, 034, 0378
N 16, 036, 046
N 17, 052
N 19, 042, 016
N 19, 042, 0318
N 20, 050, 046
N 21, 065
N 24, 065, 046
N 25, 0845
N 28, 0845, 046

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A3. Example program listings

A3.3 Harmonic Response Analysis Listing

This demonstrates the use of dynamic analysis to predict the response of a die plus transducer to a steady state forcing at a particular frequency. Response curves will be built up by repeated analysis at different frequencies but first a modal analysis is performed, for two reasons:

1) To allow the program to set up the model geometry files required for the harmonic analysis.

2) To find the natural frequencies which will define the areas of interest in the response curves.

Automatic meshing is again used for convenience in defining and developing the model, but note that in this case there is a disadvantage. Specific nodes must be chosen to have master degrees of freedom along which forces must be applied and the resulting displacements will be evaluated. Therefore the Ansys selection logic is used to find node numbers corresponding to positions on the model and specify master degrees of freedom and forces on them. The NLIST command ensures that the node numbers are recorded in

*** This couples the axial (UY) motion of node 7 to that of node 107 (effectively joining them in the axial direction).

*** geometry is now fully defined. Note that no external forces or constraints will be applied - the displacements and stresses are generated by resonance.

*** dynamic stresses are important so use "seismic excitation" to specify element formation in this part of the die outer does not lend itself to automatic element generation as above).

*** meshing is again used for convenience in defining and developing the model, but note that in this case there is a disadvantage. Specific nodes must be chosen to have master degrees of freedom along which forces must be applied and the resulting displacements will be evaluated. Therefore the Ansys selection logic is used to find node numbers corresponding to positions on the model and specify master degrees of freedom and forces on them. The NLIST command ensures that the node numbers are recorded in

...
the log file produced when the program is run so that the user can properly interpret the results.

The analysis instructions are in two stages. First the modal analysis is done, in this case using the "CHECK" mode of operation. This will create the necessary geometry files but no results (here the modal analysis results are already known). Next the program returns to the pre-processor, recalls the model and specifies the new analysis type (KAN,6 - reduced harmonic analysis). The frequencies at which the analysis is to be performed are specified using the ITER (number of iterations) and HARF (harmonic frequency range) commands. Finally the solver is used again (this time in the EXECute mode of operation) to solve the analysis.

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```
C************ START OF EXAMPLE 3 ***************
PREP7
C** Enter general purpose pre-processor PREP7
/TITL2-D MODEL ROUND DIE & TRANSDUCER (FORCED)
C** Specify analysis type (optional)
C** Now define analysis options...
KAN,2
KAY,2,1
C** Harmonic analysis initially (as before) with mode
C** extraction by frequency range.
EXMO 100, 60000
EXTRM 100, 40000
C** Frequency range for both extraction and expansion 100 Hz
C** to 40 kHz
ET 1,82
C** Element type STIF82 - 8 noded 2D isoparametric (default
C** plane stress)
TOTA 100
C** Total number of master degrees of freedom
C**
C** Now define material properties...
EX 1, 116E9
DENS 1, 4400
NUXY 1, 28
DAMP 1, 32E-9
C** Young's modulus, density, Poisson's ratio and damping
C** coefficient for material 1 (these figures for Titanium 64)
EX 2, 208E9
DENS 2, 7770
NUXY 2, 28
DAMP 2, 8E-9
C** Same properties for material 2 (Mild Steel)
C**
C** Next define model geometry...
*SET,L,130
C** Variable L is used to represent transducer length
C**
C** Keypoints...
CSYS 1
C** Start by using coord system 1 - cylindrical r,0,z
K 1 *Keypoint 1 at 0,0,0 i.e centre
K 2, 02,80
K 3, 02
K 4, 02,60
K 5, 02,120
K 6, 02,180 *Keypoints 2 to 7 arranged in circle
K 7, 02,120 *radius 0.02 at 60° intervals (inside surface)
C**
```

```
A3.4 Command File Listing

This is an example of an Ansys command file (similar to the previous examples) nested within a VAX command file. The $ character is used at the start of a VAX command line. In this case the commands specify a working directory and then run the Ansys program. Note that after the command:

$ RUN ANSYS

the format changes to Ansys commands until:

/EOF

terminates the Ansys input, after which the commands return to VAX format.

The listing is to perform a modal analysis in 3-D of a shaped die of the 3-flat type. Only half of the model is defined, with symmetrical boundary conditions fixing the "cut surface". For a 3-D model it is particularly important to use efficient modelling techniques because the number of degrees of freedom
tends to be much greater than for 2-D models. Even modelling a half die with up to 500 master degrees of freedom the accuracy is inferior to much simpler 2-D models but this type of model is necessary to properly analyse a die which is non-axisymmetric and non-uniform in the axial direction.

After the analysis a series of VAX commands is used to include the results file (FILE12.DAT) with others in a directory from which they can be retrieved by another command program for interpretation by the user.
A3.5 Command file for optimisation of outside diameter

The aim of this program was to analyse a proposed new die, find its natural frequency in the R0 mode, hence calculate the required change in die diameter and analyse the die again at this new diameter. This process would be repeated until the natural frequency was 20 kHz (to any desired degree of accuracy), whereupon the die would be analysed again to evaluate all other natural frequencies in the relevant frequency range.

The command file is applicable to almost any design of ultrasonic die. It works in conjunction with an Ansys run file giving the specific geometry and material properties. The Ansys file would be similar to example A3.2 above but with the addition of a first line to define to initial outside radius:

*SET,OR,0.0800

This first line is subsequently modified by the command file, and further commands are added at the end to process the results, extract the required resonant frequency and write that frequency out to a data file.

Working at a level above this is the main optimisation loop of the command file. The program reads the results from the data file and calculates the frequency error. If the frequency is outside the specified tolerance band then the program calculates the required correction to the outside diameter and edits the Ansys run file to include the new value before repeating the above procedure. Once the frequency is within tolerance the run-file is again edited to include additional commands to evaluate the harmonic modes as well as the axisymmetric modes. The results file from this analysis is stored for future examination by the user.

$"%%%%%%%%%%%%%%%% EXAMPLE 5 START %%%%%%%%%%%%%%%%%%%%%%%%%

$ TYPE SYS$INPUT

DIE OUTSIDE DIAMETER OPTIMISATION PROGRAM

NOTE - PROGRAM REQUIRES A TEXT FILE OF ANSYS
MODEL INPUT AS FOLLOWS:
*SET,OR,0XXX
(Replace XXX with digits. Use exactly this number of characters
- no trailing spaces etc)
*PREP7
/TITLE,XXX0000000000000000000000000XXXX...
KAN.2
Commands defining geometry, materials & options for a KAN.2
analysis. No LWRITE No AFWRITE, FINISH or EOF
$ INQUIRE FILE "Input filename for model text input:"
$ GET outside radius from model text file
$ OPEN MTFIL FILE /READ
$ READ MTFIL LINE
$ OR=F$EXTRACT(10,3,LINE)
$ Read title from MTFIL
$ IF ERROR THEN GOTO NOTTL
$ RLOOP:
$ READ MTFIL LINE
$ IF F$EXTRACT(0,5,LINE).NES."?TITL" THEN GOTO RLOOP
$ READ MTFIL LINE
$ IF TITLE.*** NO TITLE ***
$ GOTTIL:
$ CLOSE MTFIL
$ TITLF$EXTRACT(7,(F$LENGTH(LINE)-7),LINE)
$ OPEN OPTRAK TRNAME. /APPEND
$ WRITE OPTRAK TITL
$ STM="Job started +F$ELEMENT(0,".,F$TIMEQ)
$ WRITE OPTRAK STM
$ CLOSE OPTRAK
$ CREATE file OPSUB COM containing all commands for running
$ ANSYS and storing RO frequency result.
$ DELETE OPSUB.DAT *
$ Subroutine file OPSUB.COM will be created from
$ OPSUB1.TXT (first few lines which start ANSYS running),
$ FILE (the user- file which defines the model) and
$ OPSUB2.TXT (runs analysis and extracts radial-mode
$ frequency data)
$ COPY OPSUB1 TXT OPSUB.COM
$ APPEND FILE OPSUB.COM
$ APPEND OPSUB2 TXT OPSUB.COM
$ Assume that initial value of OR (outside radius of die) is
$ incorrect
$ OROK="NO"
$ $MAINLOOP
$ $ Send progress report to tracking file
$ $ TLINE=F$EXTRACT(12,5,F$TIMEQ)+" Working on OR = 0 0+OR"
$ $ OPEN OPTRAK 'TRNAME /APPEND
$ $ WRITE OPTRAK TLINE
$ $ CLOSE OPTRAK
$ $ Run model to find RO frequency
$ $ ON ERROR THEN GOTO ERANS
$ $ IF (NUMIT.EQ.MAXIT).AND.(OROK.EQS."NO") THEN GOTO .ERIT
$ $ If frequency error is less than 200 Hz then no more frequency
$ $ test runs will be made (although a small further correction
$ $ may be made to OR)
$ $ Having OROK="YES" is only way out of MAINLOOP except
$ $ for errors
$ $ Change outside radius (if freq is already within tolerance
$ $ the difference will be very small).
$ $ OR=F$STRING(F$INTEGER(OR)-FDIF/20)
$ $ and write new value to OPSUB.COM
$ $ OPEN ANFIL OPSUB.COM /READ /WRITE
$ $ READ ANFIL LINE
$ $ READ ANFIL LINE
$ $ LINE=F$EXTRACT(0,10,LINE)+OR
$ $ WRITE/UPDATE ANFIL LINE
$ $ CLOSE ANFIL
$ $ Frequency is now within tolerance (Outside Radius OK)
$ $ Send progress report to tracking file
$ $ TLINE=F$EXTRACT(12,5,F$TIMEQ)+" Tuned dimension found - OR = 0 0+OR"
$ $ TLINE=TLINE." Calculating harmonic frequencies"
$ $ OPEN OPTRAK 'TRNAME /APPEND
$ $ WRITE OPTRAK TLINE
$ $ CLOSE OPTRAK
$ $ and write optimised OR value to new OPSUB.COM
$ $ OPEN ANFIL OPSUB.COM /READ /WRITE
$ $ READ ANFIL LINE
$ $ READ ANFIL LINE
$ $ LINE=F$EXTRACT(0,10,LINE)+OR
$ $ WRITE/UPDATE ANFIL LINE
$ $ CLOSE ANFIL
$ $ Now run ansys for the final time
$ $ ON ERROR THEN GOTO ERANS
$ $ OPSUB
$ $ RENAME FILE12.DAT CHEERS.ANSYSBATCHIRESULTS.DAT
$ $ Tidy up by deleting ANSYS files
$ $ DELETE FILE%%.DAT *
$ $ Final progress report to tracking file
$ $ OPEN OPTRAK 'TRNAME /APPEND
$ $ FTIME="Job completed +F$ELEMENT(0,".,F$TIMEQ)
$ $ WRITE OPTRAK FTIME
$ $ CLOSE OPTRAK
$ $ NORMAL PROGRAM COMPLETION
A3.6 Generalised die analysis listings

The generation of general design graphs for any new ultrasonic die (described in Chapter 4) required the use of a series of programs to perform the multiple analyses and to combine and display the results. The first step was to analyse a series of simplified die models over a range of geometry variables and store the most important results. As in the previous example, the outside diameter of the die was optimised to achieve a natural frequency of 20 kHz in the desired R0 mode.

A3.6.1 Basic Parametric Programs

The program which performs the Ansys analysis of the generalised die is very similar to the example A3.2 above ie. modal analysis of an axi-harmonic model. In this case however the geometry is very simple and all important dimensions are defined by parameters so that they can be easily changed. Three versions of the basic program are used:

PHTEST6AO.COM - Command program which defines an Ansys model from a set of parameters, analyses for axisymmetric modes only and stores frequency results on a temporary file PHTEST6A.DAT.

PHTEST6B0.COM - As above, but analyses for all frequencies up to 40 kHz and up to 4th harmonic. Stores parameters, frequencies and modeshapes on two temporary files PHRES1.TXT and PHRES2.TXT. These files are added to the end of a general results file PHRES6.DAT (text format).

PHTEST6CO.COM - Dummy version of the above for combinations of materials and geometry for which a tuned OR value cannot be found (eg,
when the value of OR needs to be less than the value of SR). If used, this stores the parameter data and zero frequency and modeshape results on the general results file PHRES6.DAT.

The example below is the first of these programs. To save space the others are not reproduced here but can be easily deduced from the comment lines included here.

$1 START OF EXAMPLE 6.1
$ RUN ANSYS
$SET,OR,.0800
$OUTSIDE RADIUS -TO BE MODIFIED BY OPHBATCH
$INDEPENDE DIE VARIABLES (GEOMETRY AND MATERIALS)...
$SET,IR,.012  "inside radius"
$SET,SR,.016  "shrink-fit radius"
$SET,THIK,0.060  "thickness (across die outer)"
$SET,MAT,2  "insert (pellet) material"
$SET,MAT,2.0  "die outer (bolster) material"
$ANALYSIS VARIABLES...
$SET,MAS,130  "total number of master degrees of freedom"
$SET,NEL,6  "number of elements across die"
$PREP7  "enter Preprocessor"
$ANALYSIS VARIABLES...
$SET,MATP,288  "SYALON 101"
$SET,DEN1,3240  "density"
$SET,NU1,.23  "po"K"
$SET,MOD1,260  "NIKRO 292"
$SET,DEN2,7300  "density"
$SET,NU2,.27  "po"K"
$SET,MOD2,209  "EN41"
$SET,DEN3,7720  "density"
$SET,NU3,28  "po"K"
$SET,MOD4,74  "ALUMINIUM L16B"
$SET,DEN4,2800  "density"
$SET,NU4,.34  "po"K"
$SET,MOD5,116  "TITANIUM 318A"
$SET,DEN5,4400  "density"
$SET,NU5,28  "po"K"
$KAN2
$KAY2.1
$KEM100,40000
$KEM200,40000
$ET,1.25
$IF,MATP,NE,1,NP1  "note that material properties are"
$EX,1,MOD1*1E9  "selected from a pre-set list by"
$DEN,1,DEN1  "the parameters MATP (material"
$NUX,1,NU1  "of pellet, or die inner) and MATB"
$GO,POK  "(material of bolster, or die outer)"
$NP1
$IF,MATP,NE,2,NP2  "ERROR-UNDEFINED PELLET MATERIAL (CHECK MATP)"
$EX,1,MOD2*1E9  "POK"
$DEN,1,DEN2  "IF MATP,NE,1,NB1"
$NUX,1,NU2  "EX,2,MOD2*1E9"
$GO,POK  "DEN,2,DEN1"
$NP2
$IF,MATP,NE,3,NP3  "ERROR-UNDEFINED BOLSTER MATERIAL (CHECK MATB)"
$EX,1,MOD3*1E9  "POK"
$DEN,1,DEN3  "TOTAL MAS"
$DEN,2,DEN4  "GO,POK"
$NUX,1,NU3  "NB4"
$GO,POK  "IF MATB,NE,1,NB1"
$NP3
$IF,MATP,NE,4,NP4  "ERROR-DEFINED PELLET MATERIAL (CHECK MATP)"
$EX,1,MOD4*1E9  "POK"
$DEN,1,DEN4  "IF MATB,NE,1,NB1"
$NUX,1,NU4  "EX,2,MOD4*1E9"
$GO,POK  "DEN,2,DEN4"
$NP4
$IF,MATP,NE,5,NP5  "ERROR-DEFINED BOLSTER MATERIAL (CHECK MATB)"
$EX,1,MOD5*1E9  "POK"
$DEN,1,DEN5  "TOTAL MAS"
$DEN,2,DEN5  "GEOMETRY DEFINITION IS SIMPLE"
A3.6.2 Command Program to "Tune" each Model Die

OPHTEST6.COM - Using the list of version numbers stored on data file PHTEST6V.DAT this program analyses each set of parametric programs as follows: PHTEST6A.COM is run to find the R0 resonant frequency of this model. From this frequency a correction to OR is calculated from the formula:

\[ r = \frac{(f - 20000)}{200} \]

where \( r \) = required radius change/mm
\( f \) = resonant frequency/Hz

(This is an approximate empirical formula).

PHTEST6A.COM is modified to include this correction to OR and rerun. The new resonant frequency will be closer to 20 kHz, but again a correction will be applied. The process continues until the resonant frequency is found to be 20 kHz +/- 0.2. Then a further small correction is applied according to the above formula (no more than +/- 1 mm) and the tuned value of OR is written to PHTEST6B.COM. Running - 6B.COM generates a set of results for input parameters, modeshapes and frequencies on the results file PHRES6.DAT.

If the tuning process described above should fail, either because the frequency does not converge to 20 kHz or because the tuned value of OR is less than the known value of SR (i.e., the bolster needs to have negative thickness) then PHTEST6C.COM will be run in place of -6B. This will pass a
A3. Example program listings

set of parameter data, zero frequencies, and zero modeshapes to the results file PHRES6.DAT.

```plaintext
$OPHTEST6.COM
$ Optimises outside radius for FE model data contained in file $PHTEST6A & $B (PHTEST6B also stores results in $PHRES6.DAT)
$ Open file of version numbers to analyse
OPEN VFILE [CHEERS.PHDIPHTEST6V.DAT]
$ Create new tracking file
OPEN TFILE [CHEERS.PHTRAK.TXT] /WRITE
$ Run analysis in directory [CHEERS ANSYS2]
SET DEF [CHEERS ANSYS2]
$ Main program loop
$LOOP.
$ DELETE PHTEST6A.DAT,*
$ DELETE PHTEST6B.DAT,*
$ DELETE PHTEST6C.DAT,*
$ DELETE FILEPARM.DAT,
$ Get version numbers of PHTEST6A & B for analysis this run
ON ERROR THEN GOTO PROGCOM
$ Error here means EOF on PHTEST6V - No more versions to analyse
READ VFILE VERA
READ VFILE VERB
READ VFILE VERC
$ File 1=(CHEERS PHDIPHTEST6A COM+VERA
$ File 2=(CHEERS PHDIPHTEST6B COM+VERB
$ File 3=(CHEERS PHDIPHTEST6C COM+VERC
$ Message =F$TIMEO+ NEW JOB +F$EL(1,J,FILE1)+
+F$EL(1 J,FlLE2)
OPEN TFILE (CHEERSIPHTRAK TXT /APPEND
WRITE TFILE MESSAGE
CLOSE TFILE
$ Set maximum number of iterations before error is called
SMAXIT=5
$NUMIT=0
$ Use file 1 for model & analysis data (to find OR only)
$ Get outside radius from model text file
OPEN MFIL FILE1 /READ
READ MFIL LINE
READ MFIL LINE
OR=F$EXTRACT(94 LINE)
READ MFIL LINE
READ MFIL LINE
READ MFIL LINE
READ MFIL LINE
SR=F$EXTRACT(93.LINE)+"0"
CLOSE MFIL
$ Assume that initial value of OR (outside radius of die) is incorrect
$ OROK="NO"
$OPTLOOP: $ Run model to find R0 frequency
ON ERROR THEN GOTO ERANS
$ Delete PHTEST6A.DAT,*
@FILE1
```

```plaintext
$ Read frequency result from PHTEST6A.DAT (just created by ANSYS run)
$ ON ERROR THEN GOTO ERDR
$ OPEN RFIL PHTEST6B.DAT /READ
SLOOP:
$ READ RFIL LINE
$ IF $F$EXTRACT(2,4,LINE).NES."FREQ" THEN GOTO LOOP
$ F=F$EXTRACT(10,5,LINE)
$ CLOSE RFIL
$ ON ERROR THEN GOTO ERFH
$ MESSAGE=$F$TIMEO+$F$EXTRACT(0,12,F$LENGTH(OR),")
$ OR = 0.0")=OR
$ MESSAGE=$MESSAGE"," FREQ = "F
$ OPEN TFILE [CHEERS.PHTRAK.TXT] /APPEND
$ WRITE TFILE MESSAGE
$ CLOSE TFILE
$ ON ERROR THEN GOTO ERCAL
$ Find FDIF for frequency difference from 20 kHz
$ FDIF=20000-F$INTEGER(F)
$ and test whether it is within tolerance
$ SDIF=F$ELEMENT(1," $F$STRING(FDIF))
$ IF SDIF EGS.0 THEN ABSDF=FDIF
$ IF SDIF NES.0 THEN ABSDF=-FDIF
$ IF ABSDF.LT.200 THEN OROK="YES"
$ If frequency error is less than 200 Hz then no more frequency
$ test runs will be made (although a small further correction
$ may be made to OR)
$ Having OROK="YES" is only way out of OPTLOOP except for
$ errors
$ Change outside radius (if freq is already within tolerance the
$ difference will be very small)
$ OR=F$STRING(F$INTEGER(OR)-FDIF/20)
$ Check whether OR is now less than or approx equal to SR
$ - If so stop the analysis and store dummy data
$ F (F$INTEGER(OR) LE (F$INTEGERSR)+3)) THEN GOTO
$ TUNEF
$ otherwise write new value to PHTEST6A.COM
$ ON ERROR THEN GOTO ERAW
$ OPEN ANFIL "FILE1.README" WRITE
$ READ ANFIL LINE
$ READ ANFIL LINE
$ READ ANFIL LINE
$ IF (F$LENGTH(OR).EQ.4) THEN GOTO BIGOR
$ LINE=F$EXTRACT(0,9 LINE)+"0"+OR
$ GOTO WRITE
$BIGOR:
$ LINE=F$EXTRACT(0,9.LINE)+OR
$WRITE:
$ WRITE/UPDATE ANFIL LINE
$ CLOSE ANFIL
$ Check whether number of iterations has reached error level
$ IF (NUMIT EQ MAXIT) AND.(OROK.EQS."NO") THEN GOTO ERIF
$ If not increment iteration number
$ NNUMIT=NUMIT+1
$ Stand back to run job again (if freq was not within tolerance)
$ IF OROK.NES."YES" THEN GOTO OPTLOOP
$END.
```

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A3. Example program listings

5!
SI Frequency is now within tolerance (Outside Radius OK)

Now run ansys for the final time to produce zero$!
frequency / zero-mode data in results file
s'
$ ON ERROR THEN GOTO ERANS
$ @FILE3
SI (This will store zero parameter data by appending
PHRES6.DAT)
Si

$ ON ERROR THEN GOTO ERFH
STARTING
MESSAGE=F$TIMEQ+"
$
"+F$ELEMENT(1 TFILE2)
$ MESSAGE=MESSAGE+F$EXTRACT(O, 17-F$LENGTH(OR),"
WITH OR O.O")+OR
$ OPEN TFILE (CHEERSIPHTRAK.TXT /APPEND
$ WRITE TFILE MESSAGE
$ CLOSE TFILE

$JOBDONE:
Tidy up by deleting ANSYS files
$ DELETE FILE%%.DAT;*
$1

S ON ERROR THEN GOTO ERFH
$1
Write optimised OR value to PHTEST6B.COM
SON ERROR THEN GOTO ERWA
$ OPEN ANFIL FILEZ IREAD IWRITE
$ READ ANFIL LINE
$ READ ANFIL LINE
S IF (F$LENGTH(OR).EQ.4) THEN GOTO BIGOR2
S LINE=FSEXTRACT(O,9,LINE)s"O"+OR
$ GOTO WRITE2
$BIGOR2.
S LINE=F$EXTRACT(O,9,LINE)+OR
$WRITE2
S WRITE/UPDATE ANAL LINE
$ CLOSE ANFIL
SI
5! Now run ansys for the final time (to analyse for all modes
$1 and frequencies)
SON ERROR THEN GOTO ERANS
$ @FILEZ
$I(ThIs will also slore parameter data by appending to
PHRES6 DAT)

5!
and loop back for next version to analyse
$!
$ GOTO MLOOP
SI
$PROGCOM:
$ ON ERROR THEN GOTO END
$ MESSAGE="NORMAL PROGRAM COMPLETlOrr—"
S OPEN TFILE [CHEERSIPHTRAK.TXT /APPEND
$ WRITE TFILE MESSAGE
$ CLOSE TFILE
$ GOTO ENDPROG
$1

$1

Error calls.....
$ERCAL
$ ON ERROR THEN GOTO END
MESSAGE= — ERROR
CALCULATIONS -

DURING

$ OPEN TFILE ICHEERS]PHTRAK.TXT /APPEND
$ WRITE TFILE MESSAGE
$ CLOSE TFILE
$ GOb JOBDONE
SERDR:
$ ON ERROR THEN GOTO END
$ MESSAGE=""ERROR DURING DATA READ
$ CLOSE RFIL
$ OPEN TFILE (CHEERS]PHTRAK.TXT /APPEND
$ WRITE TFILE MESSAGE
$ CLOSE TFILE
S GOTO JOBDONE
SERANS
S ON ERROR THEN GOTO END
$ MESSAGE=""'ERROR DURING ANSYS RUN
$ OPEN TFILE (CHEERS]PHTRAK.TXT /APPEND
$ WRITE TFILE MESSAGE
S CLOSE TFILE
$ GOTO JOBDONE
SERFH
$ ON ERROR THEN GOTO END
$ MESSAGE=""ERROR DURING FILE HANDLtNG"
S OPEN TFILE [CF-4EERSIPHTRAK.TXT /APPEND
$ WRITE TFILE MESSAGE
SCLOSETFILE
$ GOTO JOBDONE
$ERWA:
$ ON ERROR THEN GOTO END
S MESSAGE="ERROR DURING WRITE ANSYS FILE"
$ OPEN TFILE [CHEERS]PHTRAK.TXT /APPEND
$ WRITE TFILE MESSAGE.
$ CLOSE TFILE
$ CLOSE ANFIL
S GOTO JOBDONE
SERIT
S ON ERROR THEN GOTO END
$ MESSAGE="ERROR- NO. OF ITERATIONS EXCEEDED
S OPEN TFILE [CHEERSJPHTRAK.TXT /APPEND
$ WRITE TFILE MESSAGE
$CLOSETFILE
$ GOTO JOBDONE
5!

SON ERROR THEN GOTO ERFH
S MESSAGEF$TIMEQ^" JOB "+F$EL(1,"]" FILE1)+"
$ MESSAGE=MESSAGE+F$EL(1,7,FILE2)+" COMPLETE"
$ OPEN TFILE ECHEERSIPHTRAK TXT IAPPEND
S WRITE TFILE MESSAGE
S CLOSE TFILE
SI

S GOTO JOBDONE
SI
SNOTUNE.
Si Tuned OR has not been found (Ic bolster thickness was going ye)
SON ERROR THEN GOTO ERFH
MESSAGE1=$ MESSAGE2=F$TIMEO+ " JOB •+F$EL(1,]" FILE1)s" S MESSAGE2=MESSAGE2+FSEL(1 ")".FILE2)+" INCOMPLETE"
$ MESSAGE3=" m TUNED OR NOT FOUND "-$ OPEN TFILE [CHEERSJPHTRAK.TXT /APPEND
$ WRITE TFILE MESSAGE1
$ WRITE TFILE MESSAGE2
$ WRITE TFILE MESSAGE3
$CLOSETFILE
5! Now write dummy OR value ( =SR) to PHTEST6C.COM
$ ON ERROR THEN GOTO ERWA
$ OPEN ANFIL 'FILE3 /READ (WRITE
$ READ ANFIL LINE
$ READ ANFIL LINE
$ IF (F$LENGTH(SR).EQ.4) THEN GOTO BIGSR
$ UNE=F$EXTRACT(O.9,LINE)+"O"+SR
$ GOTO WRITE3
$B!GSR:
$ LINE=F$EXTRACT(O9LINE)+SR
$WRITE3:
$ WRITE/UPDATE ANFIL LINE
$ CLOSE ANFIL

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A3.6.3 Command Program to Submit Multiple Analyses

PHSUB6.COM - Copies above 3 programs to temporary versions
PHTEST6A.COM - 6B.COM - 6C.COM and then modifies the parameters to
the user's specifications. Multiple copies covering a range of one or two
parameters can be produced. For each set of temporary files created, the
three corresponding version numbers are stored on data file
PHTEST6V.DAT. Finally, submits the tuning command program
OPHTEST6.COM to the batch queue for overnight processing.

```
$ PHSUB6.COM
$ $ INQUIRE ATEB 'Delete old versions (Y/N)?'
$ ATEB=F$EXTRACT(O,1 ATEB)
$ IF (ATEB NES 'Y' AND ATEB NES 'y') THEN GOTO NODEL
$ $ DEL PHTEST6A.COM
$ $ DEL PHTEST6B.COM
$ $ DEL PHTEST6C.COM
$ $ DEL PHTEST6V.DAT
$ NODEL
$ OPEN FILE PHTEST6V.DAT IWRITE
$ CLOSE FILE
$ $ SET PARAMETERS
$ $ INQUIRE DFILE 'Input data file name (PHDAT6.TXT)'
$ OPEN RDFILE 'DFILE'
$ READ RDFILE PARA1
$ READ RDFILE ITER1
$ RECI=FSINTEGER(ITER1)
$ READ RDFILE HIGH1
$ $MIDDLE
$ $ CLOSE RDFILE
$ $ OPEN FILE = PHTEST6V.DAT IWRITE
$ $ END OF MAIN EDIT
$ START LOOP
$ IF ((PARA1.NES 'IR').OR(PARA2.NES 'SR')) THEN GOTO NOTIS
$ FOR IR/SR JOB CHECK SR GREATER THAN IR
$ IF (VAL2 LE VAL1) THEN GOTO SKIP
```
A3. Example program listings

$ ! IF NOT SKIP CREATING & SUBMITTING THIS VERSION

$ NOTIS:
$ COPY PHTEST6A.COM PHTEST6A
$ COPY PHTEST6B.COM PHTEST6B
$ COPY PHTEST6C.COM PHTEST6C
$ OPEN FILE1 PHTEST6A.COM /READ IWRITE
$ OPEN FILE2 PHTEST6B.COM /READ IWRITE
$ OPEN FILE3 PHTEST6C.COM /READ IWRITE
$ SEARCH:
$ READ FILE1 LINE1 /ERROR=NOTFOUND
$ READ FILE2 LINE2 /ERROR=NOTFOUND
$ READ FILE3 LINE3 /ERROR=NOTFOUND
$ IF ($LOCATE(PARA1,LINE1).EQ.5) THEN GOTO FOUND
$ GOTO SEARCH
$ FOUND:
$ IF $LOCATE("SET",LINE1).NE.0 THEN GOTO SEARCH
$ IF $LOCATE("SET",LINE2).NE.0 THEN GOTO SEARCH
$ IF $LOCATE("SET",LINE3).NE.0 THEN GOTO SEARCH
$ LINE = "+PARA1++PREF1+F$STRING(VAL1)+"
$ LINE = F$EXTRACT(0,15,LINE)
$ WRITE AJPDATE FILE1 LINE
$ WRITE AJPDATE FILE2 LINE
$ WRITE AJPDATE FILE3 LINE
$ SEARCH2:
$ READ FILE1 LINE1 /ERROR=NOTFOUND
$ READ FILE2 LINE2 /ERROR=NOTFOUND
$ READ FILE3 LINE3 /ERROR=NOTFOUND
$ IF $LOCATE(PARA2,LINE1).EQ.5 THEN GOTO FOUND2
$ GOTO SEARCH2
$ FOUND2:
$ IF $LOCATE("SET",LINE1).NE.0 THEN GOTO SEARCH2
$ IF $LOCATE("SET",LINE2).NE.0 THEN GOTO SEARCH2
$ IF $LOCATE("SET",LINE3).NE.0 THEN GOTO SEARCH2
$ LINE = "+PARA2++PREF2+F$STRING(VAL2)+"
$ LINE = F$EXTRACT(0,15,LINE)
$ WRITE AJPDATE FILE1 LINE
$ WRITE AJPDATE FILE2 LINE
$ WRITE AJPDATE FILE3 LINE
$ CLOSE FILE1
$ CLOSE FILE2
$ CLOSE FILE3
$ GET VERSION NOS OF MODIFIED FILES AND
STORE IN PHTEST6V.DAT
$ V1=F$ELEMENT(1,"",F$SEARCH("PHTEST6A.COM"))
$ V2=F$ELEMENT(1,"",F$SEARCH("PHTEST6B.COM"))
$ V3=F$ELEMENT(1,"",F$SEARCH("PHTEST6C.COM"))
$ OPEN FILE PHTEST6V.DAT /APPEND
$ WRITE FILE V1
$ WRITE FILE V2
$ WRITE FILE V3
$ CLOSE FILE
$ ! 3 COMMAND FILES NOW EDITED
$ MESSAGE=PARA1="+PREF1+F$STRING(VAL1)"
$ MESSAGE=MESSAGE+PARA2="+PREF2+F$STRING(VAL2)" VERSIONS
$ MESSAGE=MESSAGE+V1="+V2="+V3
$ WRITE SYS$OUTPUT MESSAGE
$ !
$ ! LOOP CONTROLS
$ NOTIS:
$ VAL1=VAL1+ITER1
$ IF VAL1.LT.HIGH1 THEN GOTO START
$ VAL1=LOW1
$ VAL2=VAL2+ITER2
$ IF VAL2.LT.HIGH2 THEN GOTO START
$ IF NOL.EQS."Y" THEN GOTO LOG
$ IF AFTIM.EQS."" THEN SNA OPHTST6
$ IF AFTIM.NES."" THEN SNA /AFTER=AFTIM
$ OPHTST6
$ GOTO SUBBED
$ LOG:
$ IF AFTIM.EQS."" THEN SNA /LOG OPHTST6
$ IF AFTIM.NES."" THEN SNA /LOG
$ /AFTER=AFTIM OPHTST6
$ SUBBED:
$ ! COMMAND FILE NOW SUBMITTED
$ EXIT
$ NOTFOUND:
$ EXIT
$ CLOSE FILE1
$ CLOSE FILE2
$ CLOSE FILE3
$ TYPE SYSSINPUT
$ PARAMETER NOT FOUND - PHSUB ABORTED
$ EXIT
A3.6.4 Command Program to Read Results

PHRESREAD6.COM reads the Ansys output on the results file PHRES6.DAT, selects the useful information (i.e., important input parameters and frequencies modeshapes) converts to integer format and sends this data to a file PHRES6.INT. These files, for different material combinations have been renamed PHRES61.INT, PHRES62.INT, etc., up to PHRES611.INT.

```plaintext
$I PHRESREAD6.COM
$I Reads ANSYS output data in PHRES6.DAT
$I (including modeshape information) and writes
$I integer data to PHRES6.INT
$COPY [CHEERS ANSYS2]PHRES6.DAT
$OPEN INFILE [CHEERS ANSYS2]PHRES6.DAT
$OPEN OUTFILE [CHEERS PHD]PHRES6.INT WRITE
$OPEN TRAKFILE [CHEERS]PHTRAK.TXT WRITE
$CLOSE TRAKFILE
$SETs=0
$ON ERROR THEN GOTO END
$MAINLOOP
$READ INFILE LINE
$IF F$EXTRACT(2,2.LINE).NES 0R THEN GOTO MAINLOOP
$FLAG=-1
$WRITE OUTFILE FLAG
$VALS=F$EXTRACT(6,14.LINE)
$RETr=RETr1
$GOTO CONVERT1000
$RETr1
$WRITE OR
$WRITE OUTFILE NUM
$READ INFILE LINE
$VALS=F$EXTRACT(6,14.LINE)
$RETr=RETr2
$GOTO CONVERT1000
$RETr2
$WRITE IR
$WRITE OUTFILE NUM
$READ INFILE LINE
$VALS=F$EXTRACT(6,14.LINE)
$RETr=RETr3
$GOTO CONVERT1000
$RETr3
$WRITE SR
$WRITE OUTFILE NUM
$READ INFILE LINE
$VALS=F$EXTRACT(6,14.LINE)
$RETr=RETr4
$GOTO CONVERT1000
$RETr4
$WRITE THIK
$WRITE OUTFILE NUM
$READ INFILE LINE
$VALS=F$EXTRACT(6,14.LINE)
$RETr=RETr5
$GOTO CONVERT1000
$RETr5
$WRITE MATP
$WRITE OUTFILE VAL
$READ INFILE LINE
$VALS=F$EXTRACT(6,14.LINE)
$RETr=RETr6
$WRITE OUTFILE VAL
$READ INFILE LINE
$VALS=F$EXTRACT(6,14.LINE)
$RETr=RETr7
$WRITE OUTFILE VAL
$READ INFILE LINE
$VALS=F$EXTRACT(6,14.LINE)
$RETr=RETr8
$WRITE OUTFILE VAL
$READ INFILE LINE
$VALS=F$EXTRACT(6,14.LINE)
```

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A3. Example program listings

$ RET="RET9"
$ GOTO CONVERT1000
$ RET:
$ X3=NUM
$ READ INFILE LINE
$ VALS=FSHTRACT(6,14,LINE)
$ RET="RET10"
$ GOTO CONVERT1000
$ RET10:
$ X3=NUM
$ READ INFILE LINE
$ VALS=F$EXTRACT(6,14,LINE)
$ RET=RET10"
$ GOTO CONVERT 1000
$ RET10:
$ Y3=NUM
$ WRITE OUTFILE FREQ
$ WRITE OUTFILE X1
$ WRITE OUTFILE X1
$ WRITE OUTFILE X2
$ WRITE OUTFILE X2
$ WRITE OUTFILE X3
$ WRITE OUTFILE X3
$ WRITE OUTFILE Y3
$ WRITE OUTFILE Y3
$ Count number of iterations and loop back
$ NUMF=NUMF-1
$ IF NUMF GT 0 THEN GOTO FLOOP
$ Count number of loadsteps and loop back
$ NUMFREQ=NUMFREQ-1
$ IF NUMFREQ GT 0 THEN GOTO FREQLOOP
$ Now model parameters, 20 frequencies and 120
displacements have been
$ written - the complete results of an analysis. Loop back to
repeat until EOF
$ SETS=SETS+1
$ MESSAGE=F$TIME()$* ANALYSIS "$F$STRING(SETS)$*" 
COPIED.*
$ OPEN TRAKFILE [CHEERSJPHTRAK TXT /APPEND
$ WRITE TRAKFILE MESSAGE
$ CLOSE TRAKFILE
$ GOTO MAINLOOP
$ END
$ EOF now found - close files and stop
$ CLOSE INFILE
$ CLOSE OUTFILE
$ MESSAGE="Program complete. "$F$STRING(SETS)$*" data
sets written."
$ OPEN TRAKFILE [CHEERSJPHTRAK TXT /APPEND
$ WRITE TRAKFILE MESSAGE
$ CLOSE TRAKFILE
$ WRITE SYSSOUTPUT MESSAGE
$ EXIT
$ $ $-gradient
$ $ $ $-gradient
$ $ $-gradient
$ $ $-gradient
$ $ $-gradient

$ DEC=F$EL(0,"E",DEC)
$ DEC=DEC-"0000000000"
$ IF F$LEN(DEC),LT.EXP THEN GOTO ERR
$ DEC=F$EXP(0,EXP,DEC)
$ Find decimal part of number and shorten to sig figs
$ MULT="1"
$ COUNT=EXP
$ LOOP:
$ IF COUNT.EQ.0 THEN GOTO ENLOOP
$ MULT=MULT+"0"
$ COUNT=COUNT-1
$ GOTO LOOP
$ ENLOOP:
$ String mutt now contains multiplying factor
$ NINT=F$INT(NINT$)
$ DEC=F$INT(DEC$)
$ MULT=F$INT(MULT$)
$ Convert strings to integers
$ NUM=NINT*MULT+DEC
$ and find result
$ IF SIGN.NES." THEN NUM=-NUM
$ GOTO 'RET'
$ SIZER:
$ NUM=0
$ GOTO 'RET'
$ SINT:
$ NUM=F$INT(NINT$)
$ IF SIGN.NES." THEN NUM=-NUM
$ GOTO 'RET'
$ SERR:
$ TYPE SYSSINPUT
$ CONVERSION ERROR
$ SHOW SYM STR
$ EXIT

$ DEC=F$EL(0,"E",DEC)
$ DEC=DEC-"0000000000"
$ IF F$LEN(DEC),LT.EXP THEN GOTO ERR
$ DEC=F$EXP(0,EXP,DEC)
$ Find decimal part of number and shorten to sig figs
$ MULT="1"
$ COUNT=EXP
$ LOOP:
$ IF COUNT.EQ.0 THEN GOTO ENLOOP
$ MULT=MULT+"0"
$ COUNT=COUNT-1
$ GOTO LOOP
$ ENLOOP:
$ String mutt now contains multiplying factor
$ NINT=F$INT(NINT$)
$ DEC=F$INT(DEC$)
$ MULT=F$INT(MULT$)
$ Convert strings to integers
$ NUM=NINT*MULT+DEC
$ and find result
$ IF SIGN.NES." THEN NUM=-NUM
$ GOTO 'RET'
$ SIZER:
$ NUM=0
$ GOTO 'RET'
$ SINT:
$ NUM=F$INT(NINT$)
$ IF SIGN.NES." THEN NUM=-NUM
$ GOTO 'RET'
$ SERR:
$ TYPE SYSSINPUT
$ CONVERSION ERROR
$ SHOW SYM STR
$ EXIT

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PHREAD61.FOR is a subroutine which reads the data for modeshapes and frequencies from one of the files PHRES61.INT, PHRES62.INT, etc., as selected by the user. From the modeshape data each mode is identified, i.e., R0, R1, R2, R3, R4, T0, T1, T2, T3 and T4. The subroutine is called by the FORTRAN program PHPLOT6. and returns as results the tuned outside radius and resonant frequency in each mode for each combination of IR and SR.

SUBROUTINE READ(M,N,I,O,S,T,P,B,R0,R1,R2,R3,R4,  
T0,T1,T2,T3,T4)  
REAL R0(80),R1(80),R2(80),R3(80),R4(80)  
REAL T0(80),T1(80),T2(80),T3(80),T4(80)  
REAL QUOT  
REAL PERA1,A2,A3,A4,21,22,23  
INTEGER M(80),N(80),I(80),O(80),S(80),T(80)  
INTEGER FLAG,IR,OR SR,THIK MAS NEL, MATP,  
1 MATB,FR,FN,P,B  
INTEGER J1 N1,N2,FREQR(5),FREQT(5), FREQ,  
1 X1,X2,X3,Y1,Y2,Y3  
INTEGER FRE(4)  
FLAG=0  
J=1  
GOTO (500 510 520,530 540) B  
C NOTE IN FOLLOWING LINES PHRES6N INT  
C REFERS TO A NON-EXISTANT  
C FILE WHICH WILL CAUSE AN ERROR ON  
C OPENING THE STRUCTURE IS  
C INCLUDED TO PERMIT SUBSEQUENT  
C UPGRADING WHEN NEW DATA IS PRODUCED  
C AND NEW FILENAMES ASSIGNED (JUST  
C CHANGE APPROPRIATE FILENAME)  
501 OPEN (2,FILE='(CHEERS PHD)PHRES6N.INT'  
1, STATUS='OLD',READONLY,ERR=100)  
GOTO 560  
502 OPEN (2,FILE='(CHEERS PHD)PHRES6N.INT'  
1, STATUS='OLD',READONLY,ERR=100)  
GOTO 560  
503 OPEN (2,FILE='(CHEERS PHD)PHRES6N.INT'  
1, STATUS='OLD',READONLY,ERR=100)  
GOTO 560  
504 OPEN (2,FILE='(CHEERS PHD)PHRES6N.INT'  
1, STATUS='OLD',READONLY,ERR=100)  
GOTO 560  
505 OPEN (2,FILE='(CHEERS PHD)PHRES6N.INT'  
1, STATUS='OLD',READONLY,ERR=100)  
GOTO 560  
506 OPEN (2,FILE='(CHEERS PHD)PHRES6N.INT'  
1, STATUS='OLD',READONLY,ERR=100)  
GOTO 560  
507 OPEN (2,FILE='(CHEERS PHD)PHRES6N.INT'  
1, STATUS='OLD',READONLY,ERR=100)  
GOTO 560  
508 OPEN (2,FILE='(CHEERS PHD)PHRES6N.INT'  
1, STATUS='OLD',READONLY,ERR=100)  
GOTO 560  
509 OPEN (2,FILE='(CHEERS PHD)PHRES6N.INT'  
1, STATUS='OLD',READONLY,ERR=100)  
GOTO 560  
510 OPEN (2,FILE='(CHEERS PHD)PHRES6N.INT'  
1, STATUS='OLD',READONLY,ERR=100)  
GOTO 560  
511 OPEN (2,FILE='(CHEERS PHD)PHRES6N.INT'  
1, STATUS='OLD',READONLY,ERR=100)  
GOTO 560  
512 OPEN (2,FILE='(CHEERS PHD)PHRES6N.INT'  
1, STATUS='OLD',READONLY,ERR=100)  
GOTO 560  
513 OPEN (2,FILE='(CHEERS PHD)PHRES6N.INT'  
1, STATUS='OLD',READONLY,ERR=100)  
GOTO 560  
514 OPEN (2,FILE='(CHEERS PHD)PHRES63 INT'  
1, STATUS='OLD',READONLY,ERR=100)  
GOTO 560  
515 OPEN (2,FILE='(CHEERS PHD)PHRES64 INT'  
1, STATUS='OLD',READONLY,ERR=100)  
GOTO 560  
516 OPEN (2,FILE='(CHEERS PHD)PHRES65 INT'  
1, STATUS='OLD',READONLY,ERR=100)  
GOTO 560  
517 OPEN (2,FILE='(CHEERS PHD)PHRES66 INT'  
1, STATUS='OLD',READONLY,ERR=100)  
GOTO 560  
518 OPEN (2,FILE='(CHEERS PHD)PHRES67 INT'  
1, STATUS='OLD',READONLY,ERR=100)  
GOTO 560  
519 OPEN (2,FILE='(CHEERS PHD)PHRES68 INT'  
1, STATUS='OLD',READONLY,ERR=100)  
GOTO 560  
520 OPEN (2,FILE='(CHEERS PHD)PHRES69 INT'  
1, STATUS='OLD',READONLY,ERR=100)  
GOTO 560  
521 OPEN (2,FILE='(CHEERS PHD)PHRES30 INT'  
1, STATUS='OLD',READONLY,ERR=100)  
GOTO 560  
522 OPEN (2,FILE='(CHEERS PHD)PHRES40 INT'  
1, STATUS='OLD',READONLY,ERR=100)  
GOTO 560  
523 OPEN (2,FILE='(CHEERS PHD)PHRES69 INT'  
1, STATUS='OLD',READONLY,ERR=100)  
GOTO 560  
524 OPEN (2,FILE='(CHEERS PHD)PHRES65 INT'  
1, STATUS='OLD',READONLY,ERR=100)  
GOTO 560  
525 OPEN (2,FILE='(CHEERS PHD)PHRES66 INT'  
1, STATUS='OLD',READONLY,ERR=100)  
GOTO 560  
526 OPEN (2,FILE='(CHEERS PHD)PHRES67 INT'  
1, STATUS='OLD',READONLY,ERR=100)  
GOTO 560  
527 OPEN (2,FILE='(CHEERS PHD)PHRES68 INT'  
1, STATUS='OLD',READONLY,ERR=100)  
GOTO 560  
528 OPEN (2,FILE='(CHEERS PHD)PHRES69 INT'  
1, STATUS='OLD',READONLY,ERR=100)  
GOTO 560  
529 OPEN (2,FILE='(CHEERS PHD)PHRES30 INT'  
1, STATUS='OLD',READONLY,ERR=100)  
GOTO 560  
530 OPEN (2,FILE='(CHEERS PHD)PHRES40 INT'  
1, STATUS='OLD',READONLY,ERR=100)  
GOTO 560  
531 OPEN (2,FILE='(CHEERS PHD)PHRES69 INT'  
1, STATUS='OLD',READONLY,ERR=100)  
GOTO 560  
532 OPEN (2,FILE='(CHEERS PHD)PHRES69 INT'  
1, STATUS='OLD',READONLY,ERR=100)  
GOTO 560  
533 OPEN (2,FILE='(CHEERS PHD)PHRES69 INT'  
1, STATUS='OLD',READONLY,ERR=100)  
GOTO 560  
534 OPEN (2,FILE='(CHEERS PHD)PHRES69 INT'  
1, STATUS='OLD',READONLY,ERR=100)  
GOTO 560  
535 OPEN (2,FILE='(CHEERS PHD)PHRES69 INT'  
1, STATUS='OLD',READONLY,ERR=100)  
GOTO 560  
536 OPEN (2,FILE='(CHEERS PHD)PHRES69 INT'  
1, STATUS='OLD',READONLY,ERR=100)  
GOTO 560  
537 OPEN (2,FILE='(CHEERS PHD)PHRES69 INT'  
1, STATUS='OLD',READONLY,ERR=100)  
GOTO 560  
538 OPEN (2,FILE='(CHEERS PHD)PHRES69 INT'  
1, STATUS='OLD',READONLY,ERR=100)  
GOTO 560  
539 OPEN (2,FILE='(CHEERS PHD)PHRES69 INT'  
1, STATUS='OLD',READONLY,ERR=100)  
GOTO 560  
540 OPEN (2,FILE='(CHEERS PHD)PHRES69 INT'  
1, STATUS='OLD',READONLY,ERR=100)  
GOTO 560  
541 OPEN (2,FILE='(CHEERS PHD)PHRES69 INT'  
1, STATUS='OLD',READONLY,ERR=100)  
GOTO 560  
542 OPEN (2,FILE='(CHEERS PHD)PHRES69 INT'  
1, STATUS='OLD',READONLY,ERR=100)  
GOTO 560  
543 OPEN (2,FILE='(CHEERS PHD)PHRES69 INT'  
1, STATUS='OLD',READONLY,ERR=100)  
GOTO 560  

A3. Example program listings

1, STATUS='OLD, READONLY, ERR=100)
GOTO 560

544 OPEN (2, FILE='CHEERS.PHD') PHRES61
1, STATUS='OLD, READONLY, ERR=100)
GOTO 560

545 OPEN (2, FILE='CHEERS.PHD') PHRES68
1, STATUS=OLD, READONLY, ERR=100)
GOTO 560

560 OPEN (3, FILE='CHEERS.PHD') TEMP.DAT,
1, STATUS=NEW)

110 READ (2, * ERR=100) FLAG
IF (FLAG. NE-1) GOTO 100
READ (2) OR
READ (2) IR
READ (2) THIK
READ (2) MATP
READ (2) MATB
READ (2) MAS
READ (2) NEL

C CHECK MATERIAL NUMBERS AND CONTINUE
C READING THIS DATA SET ONLY
C IF THEY ARE AS USER SELECTED
C IF ((MATP EQ P) AND (MATB EQ B)) GOTO 139
C IF NOT THEN SKIP OVER 140 LINES OF
C UNWANTED DATA AND READ NEXT SET
CALL SKIP(140)
GOTO 110

139 DO 150 N1 = 1.5
C WRITE (3, 146)
146 FORMAT (I)
RFLAG=0
TFLAG=0
FREQR(N1) = 0
FREQT(N1) = 0
FFLAG=0

C LOOP FOR EACH MODE FOUND
C (MAX 4 EACH HARMONIC)
DO 140 N2 = 1, 4
READ (2) FREQ, X1, Y1, X2, Y2, X3, Y3
FREQ(N2) = FREQ
IF (FREQ EQ 0) GOTO 300

C CONVERT CARTESIAN DISPLACEMENTS TO
C POLAR
A4=A1+A3
IF (A4 GE 360.0) A4=A4-360
C CALCULATE ANGLE A4=A1+A3 (IN RANGE 0-360)
C
C IF (TFLAG.EQ.1) GOTO 200
C
C LOOK FOR A T-MODE IF ONE HAS NOT
C ALREADY BEEN FOUND
C IF ((Z1 EQ 0) OR (Z3 EQ 0)) GOTO 200
C IF ((Z1 EQ 0) OR (Z3 EQ 0)) GOTO 200
C CHECK Z1 & Z3 SIMILAR (WITHIN 30%) FOR (X1-EQ X2) OR (X1 EQ X3)
C CHECK X1, X3 ARE SIMILAR (WITHIN 30%)
C IF ((A2 GT 10.) AND (A2 LT 350)) GOTO 300
C CHECK A2 APPROX 0.0
C IF ((A4 GT 10.) AND (A4 LT 350)) GOTO 300
C CHECK A4 IN RANGE 0 TO 45 DEG
C IF MODESHAPE PASSES ABOVE TESTS IT IS
C CONSIDERED AN R - MODE
RFLAG=1
SET R-MODE-FOUND FLAG
REQR(N2) = FREQ
RECR(N2) = FREQ
RFLAG=0
C
C SPECIAL CASE -
C 1ST HARMONIC FREQUENCIES ARE
C SOMETIMES NOT IDENTIFIED WHEN THE
C RAND 1 FREQUENCIES ARE CLOSE
C TOGETHER - BECAUSE THE MODESHAPE
C ARE MIXED R-T. NOW TEST FOR THIS
C CONDITICN
F ((N1 N2) OR (N2 N3)) GOTO 301
F ((FFLAG EQ.2) GOTO 301
AND THAT R & T MODES HAVE NOT ALREADY
F ((FREQ(1) EQ.0) OR (FREQ(2) EQ.0)) GOTO 301
FIRST CHECK THAT THIS IS A 1ST HARMONIC
AFTER TESTING 2 MODES
C
C IF MODESHAPE PASSES ABOVE TESTS IT IS
C CONSIDERED A T - MODE
C CHECK A1 IN RANGE 0-90 DEG
C IF MODESHAPE PASSES TESTS ABOVE IT IS
C CONSIDERED A T - MODE
C
C IF (MODESHAPE PASSES TESTS ABOVE IT IS
C CONSIDERED A T - MODE
C
C IF (FFLAG.EQ.1) GOTO 200
C
C SET T-MODE-FOUND FLAG
C FREQT(N1) = FREQ
FFLAG=FFLAG+1
INCREMEN MODE-FOUND FLAG
GOTO 300
BY-PASS R MODE CHECKING

CONTINUE
C
C NOT A TORSIONAL MODE - TEST FOR RADIAL
C IF (RFLAG.EQ.1) GOTO 300
C CHECK THE RADIAL MODE HAS NOT ALREADY
C BEEN FOUND
C IF ((X1 EQ 0) OR (X2 EQ 0) OR (X3 EQ 0)) GOTO 300
QUOT=1.0*(X1-X2)/X1
IF (DABS(QUOT).GT.0.4) GOTO 300
CHECK X1, X2 ARE SIMILAR (WITHIN 40%)
QUOT=1.0*(X1-X3)/X1
IF (DABS(QUOT).GT.0.3) GOTO 300
CHECK X1, X3 ARE SIMILAR (WITHIN 30%)

IF ((A2 GT 10.) AND (A2 LT 350)) GOTO 300
CHECK A2 APPROX 0.0
IF ((A4 GT 10.) AND (A4 LT 350)) GOTO 300
CHECK A4 IN RANGE 0 TO 45 DEG
C
C IF MODESHAPE PASSES ABOVE TESTS IT IS
C CONSIDERED AN R - MODE
C
C IF MODESHAPE PASSES ABOVE TESTS IT IS
C CONSIDERED AN R - MODE
C
C IF (A4 GE 360.0) A4=A4-360
C CALCULATE ANGLE A4=A1+A3 (IN RANGE 0-360)
C
C IF (TFLAG.EQ.1) GOTO 200
C
C LOOK FOR A T-MODE IF ONE HAS NOT
C ALREADY BEEN FOUND
C IF ((Z1 EQ 0) OR (Z3 EQ 0)) GOTO 200
C IF ((Z1 EQ 0) OR (Z3 EQ 0)) GOTO 200
C CHECK Z1 & Z3 SIMILAR (WITHIN 30%) FOR (X1-EQ X2) OR (X1 EQ X3)
C CHECK X1, X3 ARE SIMILAR (WITHIN 30%)
C IF ((A2 GT 10.) AND (A2 LT 350)) GOTO 300
C CHECK A2 APPROX 0.0
C IF ((A4 GT 10.) AND (A4 LT 350)) GOTO 300
C CHECK A4 IN RANGE 0 TO 45 DEG
C IF MODESHAPE PASSES ABOVE TESTS IT IS
C CONSIDERED AN R - MODE
RFLAG=1
SET R-MODE-FOUND FLAG
REQR(N2) = FREQ
RECR(N2) = FREQ
RFLAG=0
C
C SPECIAL CASE -
C 1ST HARMONIC FREQUENCIES ARE
C SOMETIMES NOT IDENTIFIED WHEN THE
C RAND 1 FREQUENCIES ARE CLOSE
C TOGETHER - BECAUSE THE MODESHAPE
C ARE MIXED R-T. NOW TEST FOR THIS
C CONDITICN
F ((N1 N2) OR (N2 N3)) GOTO 301
F ((FFLAG EQ.2) GOTO 301
AND THAT R & T MODES HAVE NOT ALREADY
F ((FREQ(1) EQ.0) OR (FREQ(2) EQ.0)) GOTO 301
FIRST CHECK THAT THIS IS A 1ST HARMONIC
AFTER TESTING 2 MODES
C
C IF MODESHAPE PASSES ABOVE TESTS IT IS
C CONSIDERED A T - MODE
C CHECK A1 IN RANGE 0-90 DEG
C IF MODESHAPE PASSES TESTS ABOVE IT IS
C CONSIDERED A T - MODE
A3. Example program listings

```fortran
C IF (PER.GT.2.5) GOTO 301
C FREQUENCIES SEPARATED BY MORE THAN 2.5
C % ARE NOT EXPECTED
C TO CAUSE MIXED MODES
C MIXED MODES ARE NOW ASSUMED
C (IF ONE MODE HAS BEEN IDENTIFIED)
C DEFINE BOTH R & T FREQUENCIES AS THE
C AVERAGE OF FRE(1) & (2)
C FREQ=(FRE(1)+FRE(2))/2
C FREQR(N1)=FREQ
C FREQT(N1)=FREQ
C RFLAG=1
C TFLAG=1
C GOTO 301
C 286
C IF (RFLAG.EQ.1) GOTO 287
C HERE A T MODE HAS BEEN IDENTIFIED - GET
C OTHER FREQ AS FREQT
C IF (FREQT(N1).EQ.FRE(1)) GOTO 286
C FREQR(N1)=FREQ(1)
C GOTO 301
C FREQR(N1)=FREQ(2)
C GOTO 301
C TFLAG=1
C HERE AN R MODE HAS BEEN IDENTIFIED - GET
C OTHER FREQ AS FREQR
C IF (FREQR(N1).EQ.FRE(1)) GOTO 285
C FREQT(N1)=FREQ(1)
C GOTO 301
C FREQT(N1)=FREQ(2)
C GOTO 301
C END OF SPECIAL CASE
C END OF MODE-IDENTIFYING LOOP
C 301 CONTINUE
C 302 CONTINUE
C 140 CONTINUE
C 290 CONTINUE
C 150 CONTINUE
C M(J)=MAS
C N(J)=NEL
C I(J)=IR
C O(J)=OR
C S(J)=SR
C T(J)=THIK
C R0(J)=FREQR(1)
C R1(J)=FREQR(2)
C R2(J)=FREQR(3)
C R3(J)=FREQR(4)
C R4(J)=FREQR(5)
C T0(J)=FREQT(1)
C T1(J)=FREQT(2)
C T2(J)=FREQT(3)
C T3(J)=FREQT(4)
C T4(J)=FREQT(5)
J=J+1
C IF (J.LE.80) GOTO 310
C CONTINUE
C 310 CONTINUE
C 100 CLOSE (UNIT=2,STATUS=SAVE)
C CLOSE (UNIT=3,STATUS=SAVE)
J=J-1
C RETURN
END
SUBROUTINE SKIP (I)
INTEGER I,VAR
DO 90 J=1,1
READ (2,) VAR
90 CONTINUE
RETURN
END
SUBROUTINE ANGLES(X1,X2,X3,Y1,Y2,Y3,A1,A2,A3,Z1,Z2,Z3)
REAL A1,A2,A3,Z1,Z2,Z3
INTEGER X1,X2,Y1,Y2,Y3
C NORMALISE AMPLITUDE FIGURES FOR ANGLE
C A1 IN RANGE 270-90 DEG
C IF (X1.GT 0) GOTO 700
C HERE X1=0 NORMALISE FOR A1 =90 DEG
C IF (Y1.GE.0) GOTO 700
C X1=-X1
C Y1=-Y1
C X2=-X2
C Y2=-Y2
C X3=-X3
C Y3=-Y3
700 CALL CALC(X1,Y1,A1,Z1)
CALL CALC(X2,Y2,A2,Z2)
CALL CALC(X3,Y3,A3,Z3)
RETURN
END
SUBROUTINE CALC(X,Y,A,Z)
REAL A,Z,P1
INTEGER X,Y
REAL *8 ARG
P1=3.1415927
IF (X.EQ 0) GOTO 600
ARG=X*Y
Z=DSQRT(ARG)
ARG=1.0*Y/X
A=DATAN(ARG)*180./P1
IF (Y.GT 0) GOTO 630
IF (Y.LT 0) GOTO 620
A=0.0
GOTO 690
610 A=90
GOTO 690
620 A=270
630 IF (A.GE 0) GOTO 690
A=-360
GOTO 690
600 ARG=Y
Z=DABS(ARG)
IF (Y.GT 0) GOTO 610
IF (Y.LT 0) GOTO 620
A=0.0
GOTO 690
610 A=90
GOTO 690
620 A=-270
690 CONTINUE
RETURN
END
```

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A3. Example program listings

A3.6.6 FORTRAN Program to Plot Resonant Frequencies

PHPLOT6.FOR is the main program for producing the contour maps of outside radius and resonant frequencies. The user is prompted to select materials for pellet and bolster, and this information is sent to subroutine PHREAD6I which returns the outside radius and resonant frequency information. The user is then prompted for the type of contour plot required (outside radius or resonant frequency in any chosen mode) and the terminal type (Tektronix 4105 or DEC VT241). For hardcopy the chosen contour maps may be stored on file and subsequently sent to the central HP7550 colour pen plotter.

The contour maps are produced using calls to "DISSPLA" graphics routines. DISSPLA is a product of ISSCO Inc. [123]. The prototype interactive program "DISSPLA CODEBOOK" was used to automatically generate much of the FORTRAN source code.

C	 PHPLOT6 FOR
C
C	 PLOTS OUTSIDE RADIUS OR HARMONIC
C	 FREQUENCIES VS INSIDE RADIUS AND
C	 SHRINK-FIT RADIUS (DATA FROM
C	 PHREAD6 FOR)
C
C*******ISSCO CODEBOOK PROTOTYPE*****
C
C MAIN: MIMINSK
C
C MAIN FOR APPLICATIONS REQUIRING MATRIX DATA
C THE NUMBER OF MATRICES
C IS VARIABLE, AND ALL HAVE THE SAME DIMENSIONS
C RESULTING ARRAY NAMES ARE DESCRIPTIVE OF THE
C DATA REQUIREMENTS:
C RMAT1,RMAT2... RMATN WHERE N IS THE NUMBER
C OF DATA SETS
C
C APPLICATION SUBROUTINES THAT USE THIS MAIN:
C VF100, VF101, SF100, SF101, SF400, SF600,
C SF610 SF500
C
C******************************************************************************
C ARRAY IBUF USED FOR SPOOLING OUTPUT TO
C FILE J.D.DEAN 1605987
C DIMENSION RMAT1(6,13),RMAT2(6,13),
1RMAT3(6,13),IBUF(16)
REAL R0(80),R1(80),R2(80),R3(80),R4(80)
REAL T0(80),T1(80),T2(80),T3(80),T4(80)
INTEGER M(80),N(80),I(80),O(80),S(80),T(80)
INTEGER FLAG,FR,FN,X,Y,J1,J,M,P,MB
DATA NXI6I,NYI13/
C
443 IF ((MP.LT.1).OR.(MP.GT.5)) GOTO 444
444 TYPE 448
448 FORMAT ("Input material number for bolster (1-5)"
ACCEPT 9 MB
"F (MB.LT. 1) OR (MB GT 5)) GOTO 446
C
C GET DATA USING SUBROUTINE READ IN FILE
C PHREADS OBJ
CALL READ (M,1 I,OS,T,M) GOTO 446
C
C 144 FR=0
"TYPE 145
145 FORMAT ("Plot type (O-OR, R-Radial mode or T-
Torsional mode )?"
ACCEPT 11,ANS1
IF (ANS1.EQ.0) GOTO 150
IF (ANS1.EQ.R) GOTO 147
IF (ANS1.EQ.T) GOTO 146
GOTO 144
146 CONTINUE
147 FR=1
148 FORMAT ("Input mode number (0= fundamental)"
ACCEPT 149,FN
149 FORMAT ("Plot type (F)"
IF (FN.LT.0).OR.(FN.GT.4)) GOTO 146
FN=FN+1
C
C CLEAR ARRAYS RMAT1, RMAT2 & RMAT3
C CONTINUE
C******************************************************************************
CONTINUE
C
C SELECT DATA FOR PLOTTING
DO 510 J=1, J
C ELIMINATE DATA FOR WRONG NUMBER OF
C MASTERS OR ELEMENTS OR
C WRONG THICKNESS
(T(J).NE.45)) GOTO 520
C ELIMINATE SPURIOUS DATA FOR INSIDE
C RADIUS GE SHRINK-FIT RADIUS
IF (I(J).GE.S(J)) GOTO 520
C FIND PLOT COORDINATES FOR THIS DATA
POINT
X=I(J)/4-2
Y=S(J)/4-3
C ELIMINATE DATA OUT OF RANGE FOR
C REQUIRED PLOT
IF ((X.GT.NX) OR (Y.GT.NY)) GOTO 520
IF ((X.LE.0).OR (Y.LE.0)) GOTO 520
C PLACE DATA IN ARRAYS RMAT1 (OR) RMAT2
C (SR) & RMAT3 (FREQUENCY)
RMAT1(X,Y)=O(J)
RMAT2(X,Y)=S(J)
GOTO (530,540,550,560,570)
530 IF (FR EQ 0) GOTO 535
RMAT3(X,Y)=R1(J)/1000.0
GOTO 520
535 IF (FR EQ 0) GOTO 540
RMAT3(X,Y)=T1(J)/1000.0
GOTO 520
540 IF (FR EQ 0) GOTO 550
RMAT3(X,Y)=R2(J)/1000.0
GOTO 520
550 IF (FR EQ 0) GOTO 560
RMAT3(X,Y)=T2(J)/1000.0
GOTO 520
560 IF (FR EQ 0) GOTO 570
RMAT3(X,Y)=R3(J)/1000.0
GOTO 520
570 IF (FR EQ 0) GOTO 580
RMAT3(X,Y)=T3(J)/1000.0
GOTO 520
580 CONTINUE
510 CONTINUE
C
C TIDY UP AREA WHERE IR.GE.SR BY
C DUPLICATING VALUES FOR IR=SR-4
DO 620 X=1,NX
DO 630 Y=1,NY
IF (X.LT.Y+1) GOTO 640
RMAT3(X,Y)=RMRAT1(X,X)
RMAT3(X,Y)=RMRAT3(X,X)
640 CONTINUE
630 CONTINUE
620 CONTINUE
C
C TIDY UP AREA WHERE OR.EQ SR (IE. WHERE
C TUNED OR WAS NOT FOUND) BY
C DUPLICATING VALUES FOR LOWER SR
DO 650 X=1,NX
DO 660 Y=2,NY
A3. Example program listings

C ARG 1 RMAT1 REAL
C 2-DIMENSIONAL MATRIX (ARRAY) OF Z VALUES
C ARG 2-(N-3) REAL
C ADDITIONAL MATRICES, AS DESCRIBED IN ARG 1
C (FOR THIS APPLICATION, THESE ADDITIONAL MATRICES ARE IGNORED).
C ARG N-2 IXDIM INTEGER
C X DIMENSION OF MATRIX
C ARG N-1 IYDIM INTEGER
C Y DIMENSION OF MATRIX
C ARG N MATCNT INTEGER
C NUMBER OF MATRICES PASSED
C
C DATA MASTER(S) REQUIRED: MNMSK, MNMSF, MNMSKD, MNMSKF
C
C
C DIMENSION RMAT1(IXDIM,IYDIM)
C CHARACTER*20 TIT1,TIT2,TIT3
C CHARACTER*60 IIT
C COMMON IPKRAY/ IPKRAY(400)
C COMMON IMYCON/ DATMIN,DATMAX
C COMMON WORK(10000)
C USER-SUPPLIED BLANK COMMON WORK
C SPACE NEEDED BY DISSPLA FOR CONTOURING
C INTEGER MP,MB,FR,FN
C XPHY=14
C YPHY=10
C DEFINE PHYSICAL ORIGIN
C XAREA = 82
C YAREA = 65
C DEFINE SUBPLOT DIMENSIONS (AREA2D)
C
C DRAW THE CAPTION
C
CALL HEIGHT(14)
C SET CHARACTER HEIGHT
CALL DUPLX
C SET FONT TO DUPLX
CALL ALNMES(5.5)
C ALIGN MESSAGE AT CENTER, CENTER
CALL HEIGHT(12)
C SET CHARACTER HEIGHT
TIT1 = Frequency (kHz) - GOTO(400,405,410,415,420,MP)
TIT3 = Syalon 101 - GOTO(450)
TIT2 = Nikro 292 - GOTO(450)
TIT3 = Steel EN41 - GOTO(450)
TIT2 = Aluminium L168 - GOTO(450)
TIT3 = Titanium T64 - GOTO(450)
TIT2 = Aluminium L168 - GOTO(450)
TIT3 = Titanium T64 - GOTO(450)
TIT1 = TIT2/TIT3
CALL MESSAG(TIT,100,XAREA+2.,YAREA+.4)
C DRAW THE TITLE
CALL RESET(ALNMES)
CALL RESET(HEIGHT)
C RESET CHARACTER PARAMETERS
CALL BCOMON(10000)
C INFORM DISSPLA AS TO THE AMOUNT OF COMMON WORK PROVIDED
C
C GET DATA MINIMUM AND MAXIMUM
C
DATMIN=10.
A3. Example program listings

C
C ARG N  MATCNT INTEGER
C DATA MASTER(S) REQUIRED  MINMSK, MNMSF,
C MNMSKD, MINMSKF
C*****************************************************************************
C DIMENSION RMAT1(IDIM,YDIM)
C CHARACTER*20 TIT1,TIT2,TIT3
C CHARACTER*80 TIT
C COMMON (IPKRAY, IPKRAY(400))
C COMMON (MYCONV, DATMIN, DATMAX)
C STORAGE FOR DATA MIN AND MAX.FOR
C SUBROUTINE MYCON
C COMMON WORK(10000)
C USER-SUPPLIED BLANK COMMON WORK
C SPACE NEEDED BY
C DISSPLA FOR CONTOURING
C INTEGER MP, MB
C XPHY = 1.4
C YPHY = 1.0
C DEFINE PHYSICAL ORIGIN
C XAREA = 8.2
C YAREA = 6.5
C DEFINE SUBPLOT DIMENSIONS (AREA2D)
C C** DRAW THE CAPTION
C CALL HEIGHT(14)
C SET CHARACTER HEIGHT
C CALL DUPLX
C SET CHARACTER STYLE
C MAXLIN = LENEST(IPKRAY,400,80)
C INIT PACK ARRAY
C CALL LINES(Contour plot vs inside and shnrn-flt radi
C 1 S IPKRAY 1)
C CALL LINES(De thickness is 45 mm S IPKRAY 2)
C NUNES = 2
C NUMBER OF LINES IN CAPTION
C YPHY = XPHY - 1.5
C INCREDENT Y = PHYSICAL ORIGIN
C XAREA = YAREA = 1.5
C DECREMEN T Y AREA TO F T CAPI C T !
C CALL PHYSOR(XPHY, YPHY)
C DEF I NE PHYSICAL ORI G (1)
C CALL AREA2D(XAREA,YAREA)
C DEF I NE PLOT AREA (VIEWPORT)
C CALL ALIGNY(5,5)
C CAPT ON ALIGNMENT IS CENTER CENTER
C CALL STORY(IPKRAY,NUNES,XAREA, YAREA)
C PLOT CAPTION TEXT
C C** DRAW THE TITLE
C CALL DUPLX
C SET FONT TO DUPLX
C CALL ALINMES(5,5)
C ALIGN MESSAGE AT CENTER CENTER
C CALL HEIGHT (23)
C SET CHARACTER HEIGHT
C TIT1 = Outside Radius : 
C GOTO (400 405 410 415 420) MP
C 400 TIT3 = Sykon 101
C GOTO 450
C 425 TIT3 = Nickel 29Z
C GOTO 450
C 410 TIT3 = Steel EN41
C GOTO 450
C 415 TIT3 = Aluminum L168
C GOTO 450
C 420 TIT3 = Titanium Ti84
GOTO 450
450 TIT2='Sylenol 101' /'
GOTO 490
460 TIT2='Niko 292' /'
GOTO 490
465 TIT2='Steel EN41' /'
GOTO 490
470 TIT2='Aluminium L168' /'
GOTO 490
475 TIT2='Titanium T164' /'
GOTO 490
480 TIT2=TIT2/TIT2/TIT3
CALL MESSAGE(TIT1,100,XAREA/2,YAREA+4)
DRAW THE TITLE
CALL RESET('ALNMES')
CALL RESET('HEIGHT')
C
RESET CHARACTER PARAMETERS
CALL BCMONI(10000)
C
INFORM DDSPLA AS TO THE AMOUNT OF WORKSPACE PROVIDED
C
** GET DATA MINIMUM AND MAXIMUM
C
DATMIN=50
DATMAX=120
C
CALL FRAME
C
FRAME THE SUBPLOT AREA
XMIN = 12
XSTP = 4
XMAX = 32
YMIN = 16
YSTP = 8
YMAX = 64
C
DEFINE AXES MIN, STEP, MAX
C
ESTABLISH AXES LIMITS
CALL XNAME('Inside Radius IR mm's 100')
C
FORCE X AXIS TO BE DRAWN, LABEL IT
CALL YNAME('Shrink-A Ratio SR mm's 100')
C
FORCE Y AXIS TO BE DRAWN, LABEL IT
CALL GRAF XMIN1 XSTP XMAX YM1 YSTP YMAX)
C
AXES SET-UP (WINDOW)
C
BRING DDSPLA TO LEVEL 3
Z NCR = 5
C
USER-SUPPLIED Z-LEVEL INCREMENT
C
CONTOUR INTERVAL)
CALL RASPLN(25)
C
SMOOTH CONTOUR LINES
CALL CONMAX(RMAT1,JDIM,YDIM,ZDIM,ZNCR)
C
GENERATE CONTOUR LINES FROM SURFACE DATA
CALL CONLN(0 MYCCLNL 'LABELS',3,10)
C
SET CONTOUR LINE ATTRIBUTES FOR HIGHEST PRIORITY (MAJOR) LINES
DO 250 1=1,1
250 CALL CONLN1 ('MYCCLNL' 'NO LABELS' 1 9)
C
SET LINE ATTRIBUTES FOR REMAINING 1, LOWER
C
PRIORITY (MINOR) LINE
CALL CONAN(90)
C
SET MAXIMUM ANGLE OF LINE CURVATURE FOR WHICH
C
LABELS WILL NOT BE OMITTED
CALL CONU2L('LABELS' 'DRAW')
C
DRAW THE CONTOUR LINES WITH BLANKED LABELS
CALL ENDPLO(0)
C
TERMINATE THE PLOT RETURN END
C
SUBROUTINE MYCCLNL(RARRAY IARRAY CHARAY)
C

APPENDIX 4

PROPERTIES OF MATERIALS
USED FOR ULTRASONIC DIES

The following table lists material properties used in calculations for ultrasonic
dies and mountings.

Properties listed are as follows:

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>E</td>
<td>Young's Modulus of Elasticity</td>
</tr>
<tr>
<td>(\rho)</td>
<td>Density</td>
</tr>
<tr>
<td>(\nu)</td>
<td>Poisson's ratio</td>
</tr>
<tr>
<td>(v_L)</td>
<td>Sound velocity (longitudinally), (v_L = \sqrt{E/\rho})</td>
</tr>
<tr>
<td>(v_R)</td>
<td>Sound velocity (radially), (v_R = \sqrt{E/\rho(1-\nu^2)})</td>
</tr>
<tr>
<td>Q</td>
<td>Quality factor (at approx 20 kHz)</td>
</tr>
<tr>
<td>Z</td>
<td>Acoustic impedance, (Z = \sqrt{E\rho})</td>
</tr>
<tr>
<td>(\alpha)</td>
<td>Thermal expansion coefficient (at 20°C)</td>
</tr>
</tbody>
</table>

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## A4. Properties of materials

<table>
<thead>
<tr>
<th>MATERIAL</th>
<th>E (GPa)</th>
<th>ν</th>
<th>ρ (kg m⁻³)</th>
<th>Vₗ (m s⁻¹)</th>
<th>Vᵣ (m s⁻¹)</th>
<th>Q</th>
<th>α (×10⁻⁶ K⁻¹)</th>
<th>Y(Eₚ) (×10⁶ kg m⁻² s⁻¹)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aluminium L168</td>
<td>7.4</td>
<td>0.34⁷</td>
<td>2800⁴</td>
<td>5140</td>
<td>5470</td>
<td>100⁸</td>
<td>14</td>
<td>14</td>
</tr>
<tr>
<td>Titanium 6-AL, 4-V</td>
<td>114⁵</td>
<td>0.33⁵</td>
<td>4400⁵</td>
<td>5090</td>
<td>5390</td>
<td>22</td>
<td>22</td>
<td>22</td>
</tr>
<tr>
<td>Ferro-titanit Nikro 292</td>
<td>260²</td>
<td>0.27¹⁰</td>
<td>7300²</td>
<td>5970</td>
<td>6200</td>
<td>44</td>
<td>44</td>
<td>44</td>
</tr>
<tr>
<td>Ferro-titanit Nikro 128</td>
<td>294²</td>
<td>0.27¹⁰</td>
<td>6550²</td>
<td>6700</td>
<td>6960</td>
<td>31</td>
<td>31</td>
<td>31</td>
</tr>
<tr>
<td>Sylan 101</td>
<td>288³</td>
<td>0.23³</td>
<td>3240³</td>
<td>9430</td>
<td>9690</td>
<td>129</td>
<td>129</td>
<td>129</td>
</tr>
<tr>
<td>Steel EN41</td>
<td>209⁶</td>
<td>0.29¹⁰</td>
<td>7720⁶</td>
<td>5200</td>
<td>5440</td>
<td>136</td>
<td>136</td>
<td>136</td>
</tr>
<tr>
<td>Tungsten-carbide, Cobalt (typ)</td>
<td>500¹¹</td>
<td>0.23¹¹</td>
<td>12000¹¹</td>
<td>6450</td>
<td>6630</td>
<td>77</td>
<td>77</td>
<td>77</td>
</tr>
</tbody>
</table>

### Notes
- The data for Aluminium L168 is typical for aluminium alloys.
- Titanium 6-AL, 4-V is a high-strength titanium alloy.
- Ferro-titanit Nikro 292 and 128 are high-strength iron-titanium alloys.
- Sylan 101 is a ceramic material.
- Steel EN41 is a common engineering steel.
- Tungsten-carbide is a hard, wear-resistant material often used in tooling.

### Further Reading
For more detailed information on the properties of these materials, refer to the appropriate section in the Mechanics of Materials textbook or consult a specialized resource on materials science.
References:

1) Metals reference book - Smithells [89]
2) Manufacturer's data, Thyssen Edelstahlwerke AG [92]
3) Manufacturer's data sheet, Lucas Cookson Syalon Ltd [94]
4) The properties of aluminium and its alloys, Aluminium Federation [95]
5) Manufacturer's data, Reactive Metals Inc. [96]
6) Mechanical and physical properties of the BS En steels - Woolman and Mottram [90]
7) Data for pure aluminium
8) Unpublished data from Mr John Perkins (results of tests at 20 kHz)
9) Estimated from average for several tool steels
10) Private correspondance with manufacturers
11) Generic data (depends on grade)
APPENDIX 5
DIE DESIGN GRAPHS

The following pages show design graphs (contour maps) for a number of material combinations which could be used to construct an ultrasonic die. An engineer wishing to design a new ultrasonic die can consult these contour maps and get an idea of the likely problems and potential solutions before starting an accurate finite element analysis. In many cases a new ultrasonic die could be designed by reference to the contour maps without any FE analysis. For detailed descriptions and instructions for use see chapter 4.

<table>
<thead>
<tr>
<th>Outer die</th>
<th>Inner die</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) Aluminium L168</td>
<td>Syalon 101</td>
<td>345</td>
</tr>
<tr>
<td>2) Titanium Ti64</td>
<td>Syalon 101</td>
<td>348</td>
</tr>
<tr>
<td>3) Aluminium L168</td>
<td>Nikro 292</td>
<td>351</td>
</tr>
<tr>
<td>4) Titanium Ti64</td>
<td>Nikro 292</td>
<td>354</td>
</tr>
<tr>
<td>5) Steel EN41</td>
<td>Steel EN41</td>
<td>357</td>
</tr>
<tr>
<td>6) Aluminium L168</td>
<td>Steel EN41</td>
<td>360</td>
</tr>
<tr>
<td>7) Titanium Ti64</td>
<td>Steel EN41</td>
<td>363</td>
</tr>
<tr>
<td>8) Aluminium L168</td>
<td>Aluminium L168</td>
<td>366</td>
</tr>
<tr>
<td>9) Titanium Ti64</td>
<td>Aluminium L168</td>
<td>369</td>
</tr>
<tr>
<td>10) Aluminium L168</td>
<td>Titanium Ti64</td>
<td>372</td>
</tr>
<tr>
<td>11) Titanium Ti64</td>
<td>Titanium Ti64</td>
<td>375</td>
</tr>
</tbody>
</table>
A5. Die design graphs
A5. Die design graphs
A5. Die design graphs
A5. Die design graphs
A5. Die design graphs

Frequency (kHz) 

Contour plot - Titanium Ti64 / Sydon 101

Contour plot - Titanium Ti64 / Sydon 101

Contour plot - Titanium Ti64 / Sydon 101

Contour plot - Titanium Ti64 / Sydon 101
A5. Die design graphs

Contour plot vs inside and shrink-fit radii.
Die thickness is 45 mm

Resonant Frequency (kHz)
Contour plot vs inside and shrink-fit radii.
Die thickness is 45 mm
A5. Die design graphs
A5. Die design graphs
A5. Die design graphs
A5. Die design graphs
A5. Die design graphs
A5. Die design graphs

Frequency (kHz) – Aluminium L168 / Steel EN41

Frequency (kHz) – Aluminium L168 / Steel EN41

Frequency (kHz) – Aluminium L168 / Steel EN41

Frequency (kHz) – Aluminium L168 / Steel EN41
A5. Die design graphs
A5. Die Design Graphs

Resonant Frequency R1/kHz
Contour plot vs inside and shrink-fit radii.
Die thickness is 45 mm

Resonant Frequency R2/kHz
Contour plot vs inside and shrink-fit radii.
Die thickness is 45 mm

Resonant Frequency R3/kHz
Contour plot vs inside and shrink-fit radii.
Die thickness is 45 mm

Resonant Frequency R4/kHz
Contour plot vs inside and shrink-fit radii.
Die thickness is 45 mm
A5. Die design graphs

Frequency (kHz) – Titanium Ti64 / Steel EN41

Contour plot vs inside and outside radius.
Die thickness is 40 mm.

Frequency (kHz) – Titanium Ti64 / Steel EN41

Contour plot vs inside and outside radius.
Die thickness is 40 mm.

Frequency (kHz) – Titanium Ti64 / Steel EN41

Contour plot vs inside and outside radius.
Die thickness is 40 mm.
A5. Die design graphs
A5. Die design graphs
A5. Die design graphs

[Diagrams of frequency versus radius plots for different materials]

Contour plots for various dimensions and conditions.
A5. Die design graphs
A5. Die design graphs

- Frequency (kHz) - Titanium Ti64 / Aluminium Li68
- Frequency (kHz) - Titanium Ti64 / Aluminium Li68
- Frequency (kHz) - Titanium Ti64 / Aluminium Li68
- Frequency (kHz) - Titanium Ti64 / Aluminium Li68
A5. Die design graphs

Resonant Frequency T1 [kHz]
Contour plot vs inside and shrink-fit radii.
Die thickness is 45 mm

Resonant Frequency T2 [kHz]
Contour plot vs inside and shrink-fit radii.
Die thickness is 45 mm

Resonant Frequency T3 [kHz]
Contour plot vs inside and shrink-fit radii.
Die thickness is 45 mm

Resonant Frequency T4 [kHz]
Contour plot vs inside and shrink-fit radii.
Die thickness is 45 mm
A5. Die design graphs
A5. Die design graphs

- Resonant Frequency T1 kHz
  Contour plot vs inside and shrink-fit radii.
  Die thickness is 45 mm

- Resonant Frequency T2 kHz
  Contour plot vs inside and shrink-fit radii.
  Die thickness is 45 mm

- Resonant Frequency T3 kHz
  Contour plot vs inside and shrink-fit radii.
  Die thickness is 45 mm

- Resonant Frequency T4 kHz
  Contour plot vs inside and shrink-fit radii.
  Die thickness is 45 mm
APPENDIX 6
CALCULATION OF BUCKLING LOADS

This section details the calculations used to estimate the buckling resistance of alternative systems for mounting ultrasonic dies. For further discussion see chapter 5.

Euler's formula for elastic buckling of a uniform straight column with hinged ends gives the following for the critical force:

$$F = \frac{\pi^2 El^2}{l^2}$$

where
- $E =$ Young's Modulus
- $I =$ Second moment of area
- $l =$ length

Taking a rod of rectangular cross section $2.5 \times 25$ mm, $60$ mm long, made from aluminium (Young's Modulus $74$ GPa) gives:

$$I = 0.025 \times 0.0025^3 = 3.26 \times 10^{-11} \text{ m}^4$$

$$l = 0.060 \text{ m}$$

so:

$$F = 6604 \text{ N}$$

ie. This column will buckle under a load of $6.6$ kN.

For comparison, consider the formula given by Roark and Young [3] for the buckling of a thin-walled tube. Critical stress is given as:

$$\sigma' = \frac{E}{\sqrt{3} \sqrt{(1-v^2) R}}$$

where
- $t =$ wall thickness
- $R =$ mean radius of tube

$$\therefore$$
Critical force is this stress multiplied by the cross-sectional area, ie:
\[
F = \frac{2\pi E}{\sqrt{3} \sqrt{1-v^2}} t^2
\]
Evaluating this expression for an aluminium tube of 85 mm average diameter (mean radius 42.5 mm), wall thickness 2.5 mm and length 60 mm (the tubular die mounting) gives:
\[
F = 1.759 \times 10^6 \text{ N}
\]
Young advises that the critical stress actually developed is usually only 40 to 60 % of theoretical, so taking 50% the predicted buckling load is 880 kN. This is still far in excess of the critical force for the rectangular-section rod calculated above. The tube is equivalent to about 11 rods in terms of its area, but has the buckling resistance of 133 rods. This is of course a result of the curvature of the tube.

The actual buckling loads for both of the above would probably be much lower than predicted, because the ultrasonic vibrations would be expected to assist the buckling collapse. In fact, for a forming force in the range 5 to 10 kN the tube design has a very high safety factor on buckling collapse. Other possible failure mechanisms are fatigue (under the action of the vibration stresses in the hoop direction) and yield (under the action of the forming force). Fatigue failure is discussed in section 3.7. The yield force is easily calculated given the yield stress of the material (about 400 MPa for this high-strength alloy). Yield force is:
\[
F = \sigma_y \cdot 2 \cdot \pi \cdot r \cdot t
\]
Using the figures above this gives:
\[
F = 267 \text{ kN}
\]
This indicates that there is a good safety margin on yield also.
APPENDIX 7

ULTRASONIC EQUIPMENT

This section describes and lists specifications for the ultrasonic equipment used on this project, including high-power equipment for generating vibrations during can-forming, instrumentation and low-power measurement systems.

A7.1 High power systems

Three systems for generating the high power ultrasonics have been used. Initially a system based on magneto-strictive transducers, supplied by Technoform Sonics was used. This proved the principle of necking cans with ultrasonic assistance but the low efficiency inherent in this type of transducer limited its effectiveness. Later this system was used with some success for modal analysis of the ultrasonic dies by Chapman and Lucas [114], and for ESPI analysis by Shellabear and Tyrer [117], [118]. In these applications the low efficiency is an advantage because it implies a flat resonance curve, so that the system can drive the die to vibrate away from resonance. When a more effective generation system was required for process development, equipment intended for plastic welding was obtained from Kerry ultrasonics, along with extra instrumentation to assist in understanding the process. Later a second plastic welding system was obtained from Telsonic which, by using the latest electronic technology, offered higher power output and better control of the resonance mode (see section 3.3). Testing of these high-efficiency systems over a wide frequency range was not possible but some work was done to measure their performance while forming cans - see
Lucas [115] and Shellabear [119]. The components of each system will be listed separately.

A7.1 Technoform Sonics system

Generator - ENI Power Systems model EGR 1600 B (modified)

This is a variable frequency, variable output power supply, customized by Technoform Sonics to suit their transducers. Instrumentation includes a four-digit frequency indicator and moving coil wattmeter. Controls include coarse and fine frequency adjustment knobs, output level knob and overload reset switch. The main addition to the standard equipment is an automatic frequency control, operated by a frequency lock switch and bias adjustment knob (to be adjusted for maximum output).

<table>
<thead>
<tr>
<th>Specification</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nominal power supply</td>
<td>240 V / 50 Hz</td>
</tr>
<tr>
<td>Max input current</td>
<td>12 A</td>
</tr>
<tr>
<td>Working frequency</td>
<td>20000 Hz</td>
</tr>
<tr>
<td>Nominal output power</td>
<td>1600 W</td>
</tr>
<tr>
<td>Dimensions</td>
<td>500 x 440 x 250 mm</td>
</tr>
</tbody>
</table>

Transducer / Concentrator unit

The transducer part of this system comprises three magneto-strictive transducers (nickel alloy laminated cores wound with PTFE-insulated wire) brazed onto a stainless steel concentrator with a central nodal flange on which the system can be mounted.
A7.1.2 Kerry Ultrasonics system

Ultrasonic generator Kerry KS 1609

A simple high frequency power supply. All controls and instrumentation are external.

Nominal power supply  240 V / 50 Hz
Max input current  12 A
Working frequency  20000 Hz
Nominal output power  1600 W
Dimensions  330 x 180 x 620 mm

Control unit Kerry KS 457 (BB)

Dedicated control unit for the above with timer, amplitude indicator and tuning knob. The system is tuned to resonance by adjusting for maximum amplitude.

Nominal power supply  240 V / 50 Hz
Max input current  2 A
Working frequency  20000 Hz
Amplitude range  10 - 22 µ peak to peak
Timer range  0.1 - 5.9 s weld, 0 - 9 s hold
Dimensions  250 x 170 x 200 mm

Wattmeter - Sonic Systems 8314

Additional instrumentation for the Kerry ultrasonic system (or other system without power indicator), this is a true-power wattmeter with three user-selectable power ranges.

Nominal power supply  240 V / 50 Hz
Max input current  1 A
Power ranges  0 - 500, 0 - 1000 and 0 - 2000 W
Dimensions  250 x 150 x 270 mm
**Frequency Indicator - Black Star (Radio Spares)**

A general purpose frequency counter, suitable for use on ultrasonic systems. It features an 8-digit frequency display (with MHz / kHz indicator), adjustable trigger level and a switchable low frequency filter.

- **Nominal power supply**: rechargable batteries / mains adaptor
- **Frequency ranges**: 0 - 10 MHz, 0 - 100 MHz, 0 - 600 MHz
- **Gate times**: 0.1, 1.0 or 10 seconds
- **Inputs**: 50 ohm or 1 M ohm
- **Dimensions**: 220 x 90 x 230 mm

**Ultrasonic transducer**

For use with the above generator, this transducer features a titanium alloy front section, four piezo-ceramic disks and a steel back block. Connection to the generator is by four-pin plug and socket.

- **Working frequency**: 20000 Hz
- **Nominal amplitude**: 20 $\mu$ peak-to-peak
- **Max input power**: 2000 W

**A7.1.3 Telsonic system**

**Generator - Telsonic SG-22-2000**

A sophisticated power supply and control system in a single unit, featuring fully automatic frequency control (plus optional manual filter), amplitude regulation to $\pm2\%$, electronic amplitude selection from 70 to 100% of nominal (50 to 100% using external potential divider) and automatic overload and circuit protection. LED displays indicate working frequency, instantaneous load and overload.

- **Nominal power supply**: 220 V / 50 Hz (-10% / +20%)
- **Max input current**: 10 A eff.
A7. Ultrasonic equipment

**Working frequency** 20000 Hz
**Nominal output power** 2000 W eff.
**Amplitude constancy** ±2%
**Dimensions** 19" 4 HE BTH 510 x 440 x 220 mm

**Transducer - Telsonic SE 50/40-4**

Matched with the above generator, this transducer (converter) is constructed from aluminium with piezo-ceramic disks and features a protective aluminium casing mounted on a nodal flange, cooling air port and screw-connector to the high-frequency cable.

- **Working frequency** 20000 Hz
- **Nominal amplitude** 20 μ peak-to-peak
- **Max input power** 3500 W

A7.2 Low power (measuring) systems

Both the Kerry and Telsonic systems described above feature automatic control of frequency and power, to maintain resonance during operation under load and to protect the transducer from damage caused by excessive amplitude. While the user can have some influence on the generator output - power and frequency, a full evaluation of the natural frequencies would be impossible. The Technoform Sonics system offers full manual control of the generator output and it could therefore be used to determine the natural frequencies of a die (with its own concentrator-transducer unit attached). However it is not suitable to drive piezo-electric transducers for two reasons, firstly because the voltage output is wrong (magnetostrictive transducers require low voltage, high current while piezoelectrics require high voltage, low current) and secondly because it lacks the control system which limits the transducer amplitude to safe levels.
There was therefore a need for a measuring system capable of driving piezoelectric transducers at low (safe) amplitude levels with manual control of frequency and some indication of the system response. This need was satisfied by the Sonic Systems Admittance Plotter.

**A7.2.1 Sonic Systems Admittance Plotter - Model 8410**

This equipment comprises a variable frequency generator with digital frequency meter operated by manual control or automatic scan at variable rates. Output amplitude is adjustable but is normally kept at a standard (low) level. The electrical admittance (or impedance if selected) of the transducer is indicated by signal voltages at two outputs (for real and imaginary components). These should be connected to the X and Y inputs of a chart recorder so that admittance or impedance is displayed as a point on a two dimensional plot. For further details see section 6.1.2.

- Nominal power supply: 240 V / 50 Hz
- Max input current: 2 A
- Working frequency range: 15 - 29 kHz
- Frequency scan rates: 2.5 - 100 Hz / s
- Dimensions: 510 x 160 x 300 mm
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